Topic 4

Numeric and geometric patterns

Maths ideas

- Investigate and extend numeric and geometric patterns and other number patterns.
- Understand patterns with constant differences and ratios.
- Represent numeric and geometric patterns in tables.
- Represent numeric and geometric patterns in physical or diagram form.
- Describe and justify the general rules for observed relationships between numbers in own words and in algebraic language.

Key words

- **numeric sequence** – a sequence of numbers that follows a particular pattern
- **constant difference** – when you either add or subtract the same number between two consecutive terms

Describe and extend patterns

A **numeric sequence** is a sequence of numbers that follows a particular pattern. For example, there may be a **constant difference** between two consecutive terms. This means that between two consecutive terms you either add or subtract the same number.

Example

+6 +6 +6

2; 8; 14; 20; … ← In this pattern there is a constant difference of 6 between consecutive terms.

−3 −3 −3

4; 1; −2; −5; … ← In this pattern there is a constant difference of -3 between consecutive terms.

You can describe numeric sequences in your own words by first determining the constant difference.

Example

+4 +4 +4

34; 38; 42; 46; … ← Numeric sequence with a constant difference of 4, starting with 34.

−4 −4 −4

−8; −12; −16; −20; … ← Numeric sequence with a constant difference of −4, starting with −8.

EXERCISE 4.1

1. Determine the constant difference in each of these numeric patterns:

a) 1; 3; 5; 7; 9; …
b) 38; 34; 30; 26; …
c) 12; 22; 32; 42; 52; …
d) 80; 70; 60; 50; 40; …
e) 23,5; 24,5; 25,5; 26,5; 27,5; …

2 Use these rules to extend your own numeric patterns. Each pattern must have 10 terms.

a) Start with 5 and add 5.
b) Start with 529 and subtract 20.
c) Start with −10 and add 3.
d) Start with −10 and subtract −3.
e) Start with 5 and add x.

3. Describe the following numeric sequences in your own words by first determining the constant difference.

a) 21; 28; 35; 42; 49; …
b) 36; 30; 24; 18; 12; …
c) 3; 0; −3; −6; −9; …
d) −30; −15; 0; 15; 30; …
e) −10; −17; −24; −31; −38; …

In a **geometric sequence** or pattern, there is a **constant ratio** between consecutive terms. This means that between two consecutive terms you either multiply or divide by the same number.

Key words

- **geometric sequence** – a row or pattern with a constant ratio between consecutive terms
- **constant ratio** – when you either multiply or divide the same number between two consecutive terms

Example

4; 8; 16; 32; … ← In this pattern, each consecutive term is found by multiplying the previous one by 2. So this is a geometric sequence.

2; –4; 8; –16; 32; … ← In this pattern, each consecutive term is found by multiplying the previous one by –2. So this is a geometric sequence.

3; 1; $\frac{1}{3}$; $\frac{1}{9}$; … ← In this pattern, each consecutive term is found by dividing the previous one by 3. So this is a geometric sequence.

You can describe geometric sequences in your own words by first determining the constant ratio.

Example

3; 12; 48; 192; … ← This is a geometric sequence with a constant ratio of 4, starting with 3.

8; –8; 8; –8; 8; … ← This is a geometric sequence with a constant ratio of –1, starting with 8.

EXERCISE 4.2

1. Determine the constant ratio in each of these geometric sequences:
   a) 1; 2; 4; 8; 16; …
   b) 6; 18; 54; 162; 486; …
   c) 12; –12; 12; –12; 12; …
   d) 3; –6; 12; –24; 48; …
   e) 625; 125; 25; 5; 1; …
   f) 600; 300; 150; 75; …
   g) 18; 6; 2; $\frac{2}{3}$; $\frac{2}{9}$; …
   h) –20; 5; $-\frac{5}{4}$; $\frac{5}{16}$; $-\frac{5}{64}$
2. Use these rules to extend your own geometric patterns. Each pattern must have 6 terms.
   a) Start with 1 and multiply by 1.
   b) Start with 2 and multiply by –2.
   c) Start with 12 and divide by 3.
   d) Start with 25 and multiply $\frac{1}{5}$.
   e) Start with –16 and divide by –4.
   f) Start with –32 and multiply by $\frac{1}{4}$.
   g) Start with 49 and divide by 7.
   h) Start with 72 and divide by –8.
3. Describe the following geometric sequences in your own words by first determining the constant ratio.
   a) 1; 5; 25; 125; 625; …
   b) 10 000; 1 000; 100; 10; 1; …
   c) 6; –12; 24; –48; 96; …
   d) –4 096; –1 024; –256; –64; –16; …
   e) 4; –8; 16; –32; 64; …
   f) 14; 7; $\frac{7}{2}$; $\frac{7}{4}$; $\frac{7}{8}$; …
   g) 12; 4; $\frac{4}{3}$; $\frac{4}{9}$; $\frac{4}{27}$; …
   h) 0,331; 3,31; 33,1; 331; 3 310; …

Some numeric patterns have neither a constant difference nor a constant ratio.

Example

1; 1; 2; 3; 5; 8; … ← In this numeric pattern, each consecutive term is determined by the sum of the previous two terms.

1; 8; 27; 81; … ← This numeric pattern consists of cubic numbers, starting from 1.

1; 2; 4; 7; 11; 16; … ← This is a numeric pattern with consecutive natural numbers as the difference between the terms.

EXERCISE 4.3

1. Describe the following patterns. These patterns have neither a constant difference nor a constant ratio.

a) 9; 16; 25; 36; …
b) 216; 125; 64; 27; …
c) 3; 3; 6; 9; 15; …
d) 1; 11; 111; 1 111; 11 111; …
e) 3; 4; 6; 9; 13; …
f) 1; 3; 7; 13; 21; …
g) 2; 1; –1; –4; –8; …
h) 1; 2; 2; 4; 3; 6; 4; 8

2. Fill in the missing numbers in these sequences. Give reasons for your answers.

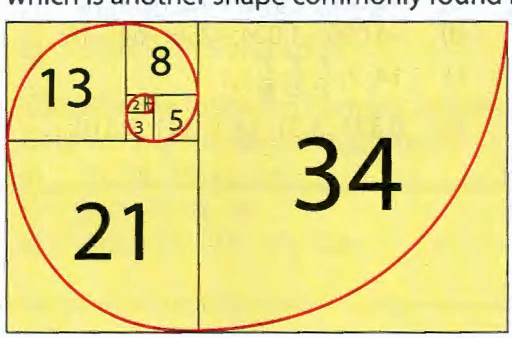
a) 5; 4; 2; –1; –5; □; –16
b) 2; 4; 8; □; 22; 32
c) 2; 1; 4; 2; 6; 3; 8; □; 10
d) –1; –8; –27; □; –125
e) 2; 1; 4; 4; 6; 9; 8; □; 10; 25

3. Make your own number patterns as described below. In each case write down the first eight terms of your number pattern. Also write down the 15th, 50th and 100th terms.

a) Make 2 different number patterns with a constant difference between consecutive terms.
b) Make 2 number patterns with a constant ratio between consecutive terms.
c) Make a number pattern that does not have a constant difference, nor a consant ratio, between the terms.

Did you know?

Look again at the first example at the top of this page. This series of numbers, discovered by the mathematician Leonardo of Pisa, is called the Fibonacci series. It is often found in nature. On many plants, the number of petals is a Fibonacci number. If you make squares with these widths, you get a spiral, which is another shape commonly found in nature.



Use tables and rules to extend patterns

A numerical pattern describes how numbers are arranged. When there is a constant difference between the numbers in a sequence, we can find a general rule that describes the pattern. This rule is often written as a **formula**. You can then substitute the given x-values into the rule or formula to find the corresponding y-values.

Key words

- **rule** – an explanation of how a pattern is arranged
- **formula** – a set of instructions for carrying out a calculation

Example

Use this rule to complete the table:

$y = -2x - 5$

x	1	2	3	4	5	6
$y = -2x - 5$	−7	−9	−11	−13	−15	−17

Substitute the x value into the equation
$y = -2(1) - 5$

By substituting different x values into the equation, we find that

$y = -2(1) - 5 \rightarrow y = -7$

$y = -2(2) - 5 \rightarrow y = -9$

$y = -2(3) - 5 \rightarrow y = -11$

$y = -2(4) - 5 \rightarrow y = -13$ and so on

EXERCISE 4.4

Use the given rules to complete the following tables.

1.

x	−1	−2	−3	−4	−5	−6
$y = -2x - 8$						

2.

x	−3	−2	−1	0	1	2
$y = -2x + 8$						

3.

q	−2	−1	0	1	10	100
$p = -5q - 4$						

4.

q	–2	–1	0	1	2	100
$p = -q - 14$						

5.

x	–2	–1	0	1	2	100
$y = x^2$						

6.

x	0	1	2	3	4	5
$y = 0{,}8x + 5$						

7.

x	3	4	5	6	7	8
$y = 0{,}2x - 5$						

8.

x	–8	–7	–6	–5	–4	–3
$y = -2{,}8x + 2$						

9.

x	0	1	2	3	4	5
$y = \frac{1}{2}x + 4$						

10.

x	–3	–2	–1	0	1	2
$y = \frac{1}{4}x + 1$						

11.

x	1	2	3	4	5	6
$y = -\frac{3}{5}x - 3$						

Challenge

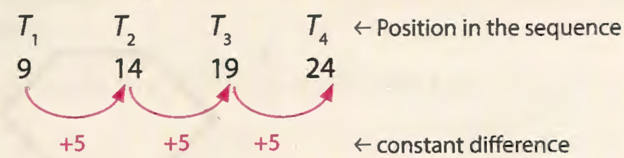
The sum $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \ldots$ is known as the harmonic series (or the sum of the terms in a sequence). If you keep adding more and more terms to this series, what do you think the sum of all the fractions will be?

Determine the rule to describe a pattern

You can describe the rule for a numeric pattern using an equation. **Terms** are the numbers or combinations of numbers and variables in a pattern.

Example

Consider the pattern 9; 14; 19; 24; …



In this pattern, the constant difference is 5. We can use equations to describe the rule as follows:

$T_1 = 5(1) + \square = 9$ **therefore** $T_1 = 5(1) + 4 = 9$

$T_2 = 5(2) + 4 = 14$

$T_3 = 5(3) + 4 = 19$

$T_4 = 5(4) + 4 = 24$

Notice how the position of the term corresponds to the number in the brackets:

$T_{15} = 5(15) + 4 = 79$

The n^{th} term of the pattern is $T_n = 5(n) + 4$ where we substitute the term number and the number in brackets with ***n***.

EXERCISE 4.5

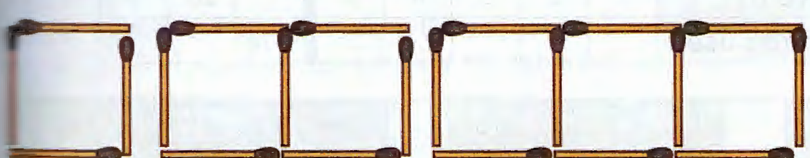
Use equations to describe the rule and the n^{th} term for the following patterns:

1. 10; 14; 18; 22; …
2. 34; 26; 18; 10; …
3. 17; 11; 5; –1; –7;…
4. 4; 11; 18; 25; …
5. –35; –27; –19; –11; …

Key words

- **term** – a number, or a combination of a number and a variable in a numerical pattern or mathematical expression

You can use a sketch to determine the constant difference and then use equations to describe the rule in terms of *n*. Then you can complete the table.



Pattern 1 Pattern 2 Pattern 3

In this pattern, the constant difference = 3 matchsticks. So, using equations, we can describe the rule as follows:

$T_1 = 3 \times 1 + 1 = 4$

$T_2 = 3 \times 2 + 1 = 7$

$T_3 = 3 \times 3 + 1 = 10$

$T_4 = 3 \times 4 + 1 = 13$

$T_n = 3 \times n + 1 = 3n + 1$

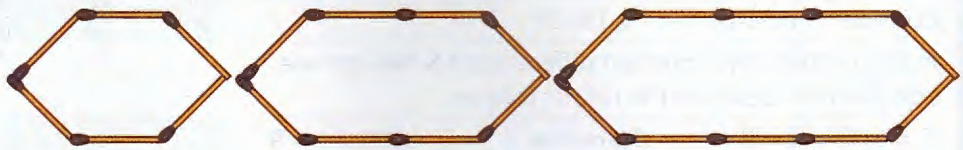
Once we have described the rule, we can use it to complete the table:

Term (pattern)	1	2	3	4	5	14	15	n
No of matches used	4	7	10	13	16	43	46	$3n + 1$

EXERCISE 4.6

Use the given sketches of matchsticks to draw the next pattern in the sequence. Then use equations to describe the rule and determine the n^{th} term. Finally, complete the tables that follow.

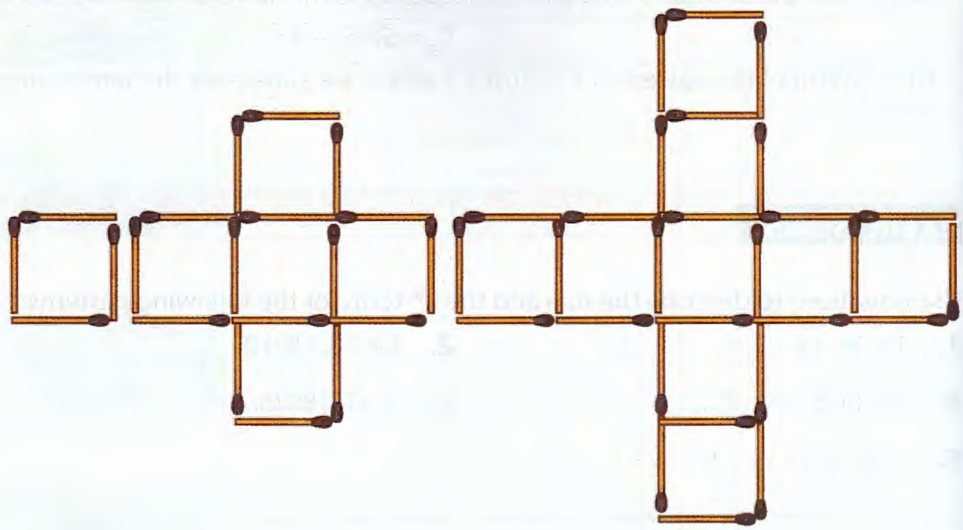
1.



Pattern 1 Pattern 2 Pattern 3

Term (pattern)	1	2	3	4	5		18	n
No of matches used						34		

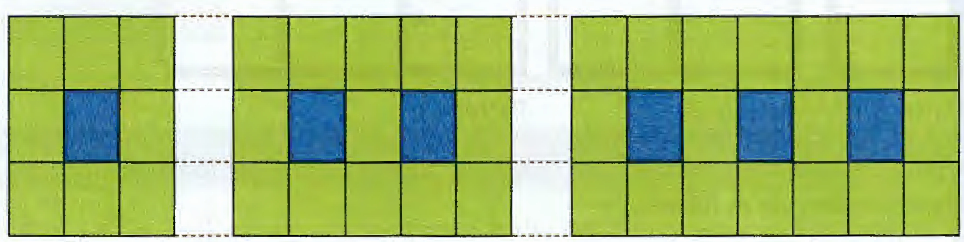
2.



Pattern 1 Pattern 2 Pattern 3

Term (pattern)	1	2	3	4	5		20	n
No of matches used						208		

3.



Pattern 1 Pattern 2 Pattern 3

Term (pattern)	1	2	3	4	5		20	n
No of green squares						73		

Revision

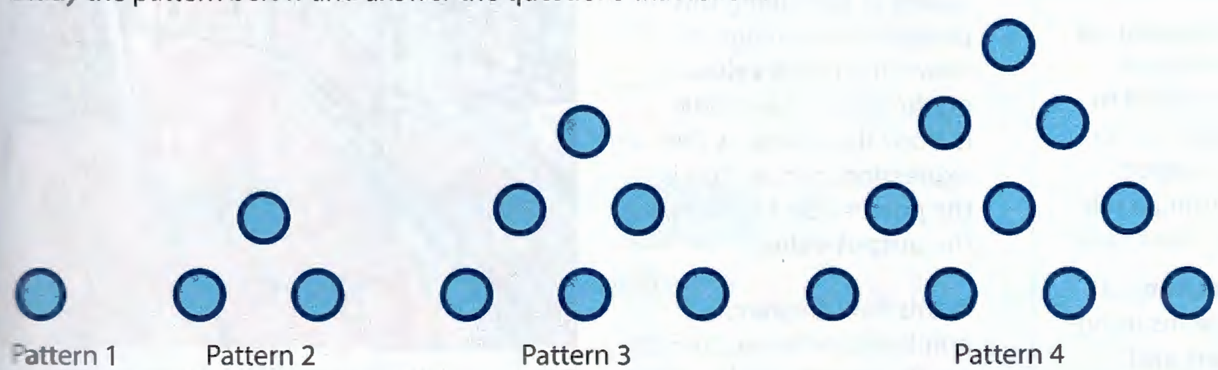
1. Study the following numeric pattern and answer the questions that follow. 3; 10; 17; 24; 31; …
   a) Determine the constant difference of the pattern. (1)
   b) Write down the next term in the sequence. (1)
   c) Write down a rule in your own words to describe this pattern. (1)
   d) Use your rule to determine the 11th term of this pattern. (1)

2. Study the following geometric pattern and answer the questions that follow. 3; –6; 12; –24; 48; …
   a) Determine the constant ratio of the pattern. (1)
   b) Write down the next term in the sequence. (1)
   c) Write down a rule in your own words to describe this pattern. (1)
   d) Use your rule to determine the 10th term of this pattern. (1)

3. Describe the following patterns in your own words, and write down the next term for each sequence:
   a) 1; 4; 9; 16; 25; …   b) 1; 8; 27; 64; 125; … (4)
   c) 1; 2; 4; 7; 11; 16; …   d) 0; 1; 1; 2; 3; 5; 8; 13; … (4)

4. Describe the following patterns in your own words, and then extend each pattern by the next 3 terms in the sequence.
   a) 1; 3; 5; 7; 9; …   b) 2; 1; $\frac{1}{2}$; $\frac{1}{4}$; … (4)
   c) 6; 3; 0; –3; –6; …   d) 1; 2; 4; 8; 16; … (4)
   e) 4; 44; 484; 5 324; …   f) 1; 3; 6; 10; 15; … (4)

5. Study the pattern below and answer the questions that follow.



   a) Draw the next pattern in the sequence. (1)
   b) Use the pattern to complete the table below. (2)

Pattern number	1	2	3	4	9
Number of dots	1	3	6		

6. Use the given rules to complete these tables.

   a)

x	2	1	0	–1	–7
$y = 5x$	10	5			

(2)

   b)

x	1	0	–1	–2	–10
$y = –2x + 1$	–1	1	3		

(2)

7. Determine the constant difference in this numeric pattern.
   a) 5; 9; 13; 17; 21; … (1)
   b) Now use equations to determine the n^{th} term for the pattern. (4)

Total marks: 40