

## Maths ideas

- Write equations to describe problem situations.
- Analyse and interpret equations that describe a given situation.
- Solve equations by inspection.
- Solve equations using laws of exponents

## Key words

- **algebraic equation** – an equation containing variables and constants
- **root** – the number that makes an equation sentence true; the solution to the equation
- **linear equation** – an equation of degree one; there is one possible solution

## Solve equations

To solve problems in algebra, you can set up an **algebraic equation**. This is an equation that you can solve, based on information that you are given. The solution to the equation is also called the **root** of the equation.

### Example

What must be added to 79 to get an answer of 123?

#### Answer

You can write this equation in different ways:

$$79 + y = 123 \text{ or } 123 - y = 79 \text{ or } y = 123 - 79$$

All three equations give the same answer:  $y = 44$ .

The letters  $x$  and  $y$  are often used in Mathematics in place of the unknown quantity, but any letter may be used. A **linear equation** is one where there is one solution to the equation. The degree of the equation is one.

You can solve simple equations by inspection or substitution. You know that 5 multiplied by 90 is equal to 450. The answer is 10 more than 5 multiplied by 90, so you can substitute  $x = 92$  into the equation to see whether it is the correct answer:  $5(92) = 460$  so  $x = 92$  is the correct answer.

When solving equations, use the inverse operations to simplify your equation and find the answer.

### Example

$$\text{Solve } 3x - 4 = 68$$

#### Answer

**Step 1:** Add 4 to both sides of the equation to isolate the variable term:  $3x = 72$

**Step 2:** Divide both sides of the equation by 3 to find the value of  $x$ :  $x = 24$

**Step 3:** Check that your answer is correct by substituting  $x = 24$  into the original equation:  $3(24) - 4 = 68$

If there are brackets in an equation, expand the equation by multiplying out the brackets. Then follow the steps above to solve.

### Example

$$\text{Solve the equation } 3(x - 2) = 12$$

#### Answer

**Step 1:** Multiply out the bracket:  $3x - 6 = 12$

**Step 2:** Add 6 to both sides of the equation to isolate the variable term:  $3x = 18$

**Step 3:** Divide both sides of the equation by 3 to find the value of  $x$ :  $x = 6$

**Step 4:** Check that your answer is correct by substituting  $x = 6$  into the original equation:  $3(6 - 2) = 12$

When there are variable terms on both sides of the equation, get all the variable terms on the same side of the equals sign and then add like terms.

### Example

Solve the equation  $5(x - 1) = x + 3$

#### Answers

**Step 1:** Multiply out the bracket:  $5x - 5 = x + 3$

**Step 2:** Subtract  $x$  from both sides so that variable terms are together:

$$5x - x - 5 = 3$$

**Step 3:** Add 5 to both sides of the equation to isolate the variable term:  $4x = 8$

**Step 4:** Divide both sides of the equation by 4 to find the value of  $x$ :  $x = 2$

**Step 5:** Check that your answer is correct by substituting  $x = 2$  into the original equation:  $5(2 - 1) = 2 + 3 = 5$

When there is a fraction in an equation, first multiply both sides by the lowest common denominator (LCD) to get rid of the fractions and then continue working through the steps to solve the equation.

### Example

Solve the equation:  $\frac{x}{3} + \frac{2x-1}{4} = 1$

#### Answer

**Step 1:** Multiply **both** sides by the LCD 12:  $4x + 3(2x - 1) = 12$

**Step 2:** Multiply out the bracket:  $4x + 6x - 3 = 12$

**Step 3:** Add the like terms:  $10x - 3 = 12$

**Step 4:** Add 3 to both sides of the equation to isolate the variable term:  $10x = 15$

**Step 5:** Divide both sides of the equation by 10 to find the value of  $x$ :  $x = \frac{3}{2}$

**Step 6:** Check whether your answer makes sense.

## EXERCISE 9.1

- What number divided by 12 gives an answer of 72?
  - What number must be added to 8 to get an answer of  $-23$ ?
  - What number when multiplied by 15 gives  $-90$ ?
- Solve the following equations:
 

a) $x + 7 = 15$	b) $m - 2 = 17$	c) $5y = 3y + 12$
d) $13p - 6 = 33$	e) $5a - 22 = a - 2$	f) $n - 4 = -6$
- Solve for  $x$ :
 

a) $\frac{x}{2} = 25$	b) $\frac{x}{-3} = 12$
c) $\frac{x}{7} + 2 = -5$	d) $\frac{x-6}{2} = -1$
e) $\frac{2x-3}{2} - \frac{3x+1}{4} = 1$	f) $\frac{2(x-1)}{3} - \frac{3}{4} = \frac{3(2x+3)}{2} - 3$
- Solve for  $x$ :
 

a) $3(x - 2) = 2(x - 4)$	b) $2(2x - 4) = 3(3x + 4)$
c) $6 - 2(x - 1) = 4x - 16$	d) $2(2x + 9) - 2(x + 3) = x + 11 - 5(4 - x)$
- Solve the following equations:
 

a) $4(7x + 6) = 3(9x + 8)$	b) $\frac{x}{6} + x + 2 = x + \frac{5}{2}$
c) $\frac{x+3}{4} - \frac{x}{2} = \frac{x+2}{8} - 1$	d) $\frac{3}{4}(3x - 5) - \frac{9}{4} = \frac{1}{2}(2x + 4)$

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### Did you know?

In Mathematics, the difference between an expression and an equation is that you can simplify expressions, but you cannot solve them. Equations have equals signs (=) and can be solved.

To summarise, when solving linear equations:

- Get rid of fractions by multiplying both sides of the equation by the LCD.
- Multiply out brackets.
- Get variable terms on one side and constants on the other side of the equal sign.
- Simplify where necessary.
- Lastly, divide both sides of the equation by the coefficient of the variable term.
- Check that your answer makes sense.

## Equations with exponents

When solving equations with exponents, the variable may be a part of the base or in the exponent part of a power. If the unknown is part of the exponent, make the bases the same on both sides of the equation, if possible. The exponents are then equal and you can drop the bases to solve the equation.

### Example

Solve for  $x$  if  $3(5^x) - 1 = 374$

**Answer**

$$3(5^x) = 375$$

← add 1 to both sides of the equation

$$5^x = 125$$

← divide both sides of the equation by 3

$$5^x = 5^3$$

← write the base the same on both sides

$$\therefore x = 3$$

If the unknown is part of the base, you need to isolate the variable term. Then see whether you can make the exponents equal on both sides of the equation.

### Example

Solve for  $x$  if  $2x^5 + 9 = 73$

**Answer**

$$2x^5 = 73 - 9$$

← subtract 9 from both sides of the equation to isolate the variable term

$$2x^5 = 64$$

$$2x^5 \div 2 = 64 \div 2$$

← divide both sides by the coefficient of the variable term

$$x^5 = 32 = 2^5$$

← make the exponents equal on both sides of the equation

$$\therefore x = 2$$

## EXERCISE 9.2

1. Solve for  $x$ :

a)  $2^{x+1} = 16$

b)  $3^x = \frac{1}{27}$

c)  $2^{-x} = 32$

d)  $8^x = 16$

e)  $5^{2x-1} = 0,008$

f)  $10^x = 0,0001$

2. Solve for  $x$

a)  $x^{\frac{3}{4}} = 8$

b)  $2x^{\frac{2}{3}} = 32$

c)  $\frac{1}{3}x^5 = 81$

d)  $2x^3 - 4 = 246$

3. If  $5^{-2} \times 5^x = 1$ , solve for  $x$ .

4. The product of the square of a number and the cube of the same number is equal to 32. Find the number.

5. Find the value of  $x$  if  $\frac{a \times a^3}{\sqrt{a^2}} = a^x$

## Solve word problems

When solving word problems, let the unknown quantity be  $x$ . Use mathematical symbols to write an equation describing the problem.

Solve the equation and write your answer in words.

Check that your solution makes sense.

Equations are useful for solving practical problems, for example those that involve money or measurement. Leave out any units describing money or length when solving equations. Remember to write the final answer with the correct units.

This table shows symbols that are commonly used in equations, and their meanings in words:

Algebraic symbol	English translation	Algebraic symbol	English translation
=	is equal to	≠	is not equal to
+	sum (increase)	-	difference (decrease)
×	product	÷	quotient

### Example

- If one side of a square is extended by 6 m and the adjacent side is reduced by 4 m a rectangle with a perimeter of 84 m is formed. Calculate the area of the original square.
- A boy has three times as many R2 coins as R1 coins. The value of his money is R28. How many coins of each type does he have?

#### Answers

- Let a side of the square =  $x$  metres

The sides of the rectangle are  $(x + 6)$  m and  $(x - 4)$  m

Perimeter of rectangle =  $2(x + 6) + 2(x - 4) = 84$  m

$$2x + 12 + 2x - 8 = 84$$

← multiply out the brackets

$$4x + 4 = 84$$

← add like terms

$$4x = 80$$

← subtract 4 on both sides of the equation

$$x = 20$$

← divide both sides by 4

The area of the original square =  $20^2 = 400 \text{ m}^2$

Check your answer:  $2(26) + 2(16) = 84$

- Let the number of R1 coins =  $x$  and the number of R2 coins =  $3x$ .

$$3x(2) + x(1) = 28$$

← write an equation without units

$$7x = 28$$

← add like terms

$$x = 4$$

← divide both sides by 7

He has four R1 coins and twelve R2 coins. ← write the answer in words

Check your answer:  $12(2) + 4(1) = 28$

let number of R1 coins be  $x$

$$3x(2) + x(1) = 28$$

$$6x + x = 28$$

$$7x = 28$$

$$x = 4$$

4 = R1 coins

12 = R2 coins

When solving problems, it helps to draw a table showing the information given.

### Example

The sum of the ages of two teachers is 70 years. In 10 years time, Mr A will be twice as old as Mr B was 8 years ago. How old are Mr A and Mr B now?

#### Answer

Show all the necessary information in a table:

	8 years ago	Now	10 years time
Mr A	$x - 8$	$x$	$x + 10$
Mr B	$62 - x$	$70 - x$	$80 - x$

- $x + 10 = 2(62 - x)$  ← write an equation describing the situation  
 $x + 10 = 124 - 2x$  ← multiply out the bracket  
 $x + 2x = 124 - 10$  ← get variables on LHS and constant terms on RHS  
 $3x = 114$  ← simplify by adding like terms  
 $x = 38$  ← divide both sides by 3

The teachers are 32 and 38 years old.

### EXERCISE 9.3

- A farmer sells a certain number of chickens at R20 each and double the number of chickens at R30 each. If his total income is R1 280, how many chickens did he sell at each price?
- A father is now 38 years older than his son. Ten years ago he was twice as old as his son. How old are the father and son now?
- Chad is training for a swimming gala by swimming 40 lengths a day. He worked out that if he doubled the lengths he has swum so far and then added six, he would have swum the 40 lengths.
  - Write an equation using the variable  $x$  showing the information given.
  - Solve the equation to find the value of  $x$ .
- Find three consecutive numbers that add up to  $-54$ .
- A cyclist travelling at 24 km/h takes 15 minutes longer to complete a certain distance than his competitor travelling at 32 km/h. Calculate the distance covered.
- Find two numbers such that the sum of three times the first and twice the second is 40. The first number, increased by two and then multiplied by 5, is also 40.
- In three years time a dog will be as old as his owner was four years ago. Their present ages total 13 years. Find the present age of the dog and his owner.
- A group of six learners have a bag of  $x$  sweets. If there were two more sweets in the bag, there would be enough for each learner to have 12.
  - Which of these equations represents this information?
    - $6x + 2 = 12$
    - $\frac{x+2}{6} = 12$
    - $\frac{x-2}{6} = 12$
  - How many sweets are in the bag?
- A batsman scored 84 runs off fours, sixes and singles. He hit an equal amount of fours and sixes and twice as many singles as fours. How many of each did he hit?
- In a multiple choice test, two marks are given for each correct answer and one mark is deducted for every incorrect answer. If a pupil answers 20 questions and gets 25 marks, how many correct answers were there?

### Challenge

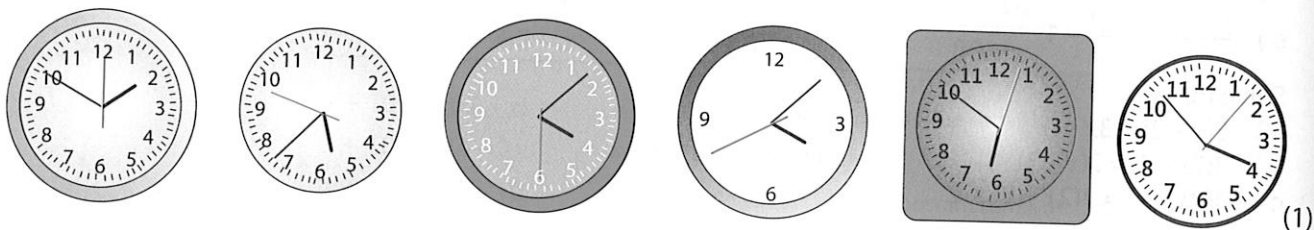
A learner misses the train by 2 minutes if he walks to the station at an average speed of 4 km/h. If he runs at an average speed of 8 km/h, he arrives at the station 5 minutes and 30 seconds before the train departs. Calculate the distance to the station.

1. A number, increased by four times the number, is 35. Find the number. (2)
2. Solve the following equations:
  - a)  $6a + 2 = 74$  (2)
  - b)  $9n - 3 = -66$  (2)
  - c)  $\frac{b}{4} = 5$  (1)
  - d)  $\frac{2x-1}{3} = -5$  (3)
  - e)  $\frac{y+1}{2} = \frac{2y+1}{5} - 1$  (4)
3. Solve for  $x$ :
  - a)  $5(x-2) = 3x-4$  (3)
  - b)  $3(2x-7) = 2(x-3)$  (3)
  - c)  $4(2x-1) = 2(2x+3) + 2$  (4)
4. Solve for  $x$ :
  - a)  $3^{x-1} = 81$  (2)
  - b)  $4^{-x} = 32$  (2)
  - c)  $2^x = 0,125$  (2)
5. The product of the square of a number and the cube of the same number is equal to 243. Find the number. (3)
6. The sum of two numbers is 12. The product of the smaller number and 3 is  $-6$ . Find the numbers. (3)
7. The length of a rectangle is 4 cm more than its width. If its perimeter is 20 cm, find the width. (3)
8. A father is three times as old as his son. In ten years' time their combined ages will be 68. How old are they now? (4)
9. Alan keeps fit by cycling each night. He warms up for 5 minutes and then cycles several laps. Each lap takes him 2 minutes to complete.
  - a) If he cycles  $x$  laps, write down an equation for the total time he needs, using the information given. (1)
  - b) Calculate how many laps he cycles, if he takes 45 minutes altogether. (3)
10. The ratio of  $\hat{A}$ ,  $\hat{B}$  and  $\hat{C}$  in triangle ABC is 2 : 3 : 4. Calculate the size of each angle. (4)
11. Two hundred and forty people attended a concert. The tickets cost R60 per adult and R40 per child. If the concert raised R12 800, how many adults and how many children attended the concert? (4)

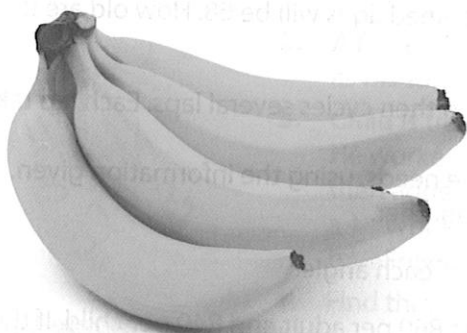
**Total: 55 marks**

# Formal Assessment Exemplar: Test

- From the set of numbers  $\sqrt{7}$ ;  $-11,35$ ;  $-\sqrt{25}$ ;  $0,13131313\dots$ ;  $\pi$ ; choose:
  - two rational numbers (1)
  - two irrational numbers. (1)
- Write three equivalent fractions for  $-2\frac{5}{6}$  (1)
- Calculate the total cost of six clocks costing R79,95 each. (1)



- A box of cereal weighs  $\frac{3}{4}$  kg. Calculate the mass of a carton containing two dozen boxes of cereal. (1)
- Olympus Mons rises about 22 km into the Martian atmosphere. Mount Everest is about 8 800 m high. How many times higher than Mount Everest is Olympus Mons? (2)
- Thabo bought some bananas. On Monday, he ate  $\frac{2}{5}$  of them, on Tuesday  $\frac{15}{50}$  and on Wednesday  $\frac{6}{30}$ . What fraction of bananas remained? (3)



$$\begin{array}{r}
 \frac{2}{5} \times 30 = 12 \\
 \frac{15}{50} \times 30 = 9 \\
 \frac{6}{30} \times 30 = 6 \\
 \hline
 12 + 9 + 6 = 27 \\
 30 - 27 = 3 \\
 \frac{3}{30} = \frac{1}{10}
 \end{array}$$

- Calculate:
  - $\sqrt{0,35 - 0,1}$  (1)
  - $\sqrt[3]{0,080 - 0,016}$  (1)
- It takes 10 men 6 days to cut down 56 trees in the forest. Calculate how long it will take 15 men to do the same job. (1)
- Calculate the following:
 

	trees	days	men
a)	56	6	10
b)	56	4	15

  - $3^1 \times 7^0 \times 3^{-1}$  (2)
  - $8^{15} \div 8^{13}$  (2)
  - $\frac{2^0 \times 2^{-2} \times 3^2}{2^{-3} \times 3^{-1} \times 3^3}$  (4)

10. Simplify the following:
- a)  $2ab^2 \times 4a^2b^{-1} \times 3a^{-2}b^2$  (3)
- b)  $(5a)^3 \div 5a^{-1} \times 5^{-2}a^2$  (3)
11. Solve  $6^x + 1 = 217$  (2)
12. Calculate:
- a)  $(2,46 \times 10^{-6}) \times (4,01 \times 10^{-5})$  (3)
- b)  $(9,12 \times 10^{-14}) \div (3,2 \times 10^{-3})$  (3)
- c)  $\frac{(9,35 \times 10^6) \times (1,5 \times 10^{-8})}{3,4 \times 10^{-6}}$  (4)
13. If the mass of Earth is  $5,98 \times 10^{24}$  kg and the moon has a mass of  $7,36 \times 10^{22}$  kg, calculate the difference in their masses. Write your answer in scientific notation. (2)



14. Calculate the number of seconds in a day. Write your answer in scientific notation. (2)

15. **Lounge suite  
for sale**

**ONLY R600 deposit!**

24 months to pay.

Cash price R 9 500

Study the lounge suite advert above.

- a) Calculate what percentage of the cash price the deposit is.  
Write your answer correct to one decimal place. (2)
- b) Mr Mathabo decides to pay the deposit of R600 and to pay the balance over 24 months at 17% p.a. simple interest. Calculate his total payment at the end of the second year. (2)
- c) How much less would he pay for the lounge suite if he pays cash? (1)
- d) Inflation is estimated at 6,5% per annum. What will the cash price of this lounge suite be in 2 years time? Round off your answer to the nearest rand. (2)

**Total: 50 marks**