Key words

- **multiple** – the product of two natural numbers
- **lowest common multiple (LCM)** – the lowest multiple that is common to two or more numbers

Lowest common multiples (LCM)

A **multiple** is the product of two natural numbers. For example, 48 is a multiple of 12 and 4 because $12 \times 4 = 48$. The number 24 is also a multiple of 12 and 2 because $12 \times 2 = 24$. Every natural number has an infinite number of multiples.

You can also find the **lowest common multiple (LCM)** of two or more numbers. To do this, you list as many multiples as you need until you find one that is common to both or all of the numbers.

Example

Find the LCM of 12 and 9.

Answer

The multiples of 12 are: 12; 24; **36**; 48; 60; 72; 84 …

The multiples of 9 are: 9; 18; 27; **36**; 45; 54; 63; 72; 81; 90 …

So the LCM of 12 and 9 is 36.

Instead of writing out a long list of multiples for each number, you can use the prime factor method to find the LCM.

Example

Find the LCM of 6, 8 and 15.

Answer

Write each number as a product of its prime factors:

$6 = 2 \times 3$ $\quad 8 = 2 \times 2 \times 2$ $\quad 15 = 3 \times 5$

$LCM = 2 \times 2 \times 2 \times 3 \times 5 = 120$

All the prime factors of each number are in the product. Any common factors are only used once.

EXERCISE 1.5

1. Find the HCF for these sets of numbers.
   a) 21 and 84 b) 108 and 270
   c) 30, 210 and 700 d) 195 and 330
2. Find the LCM for these sets of numbers.
   a) 24 and 90 b) 18 and 48 c) 90, 105 and 150
3. Mr Abrahams is able to divide the learners in his Mathematics class into equal groups of 2, 5 or 6 without leaving any learners out. What is the smallest possible number of learners in the class?
4. In a village, the bell rings every 30 minutes at the Junior School and every 40 minutes at the High School. Both schools start lessons at 8:00 a.m.
   a) At what times between 8:00 a.m. and 2:30 p.m. do the bells both ring at the same time?
   b) If the High School bell rings every 45 minutes, how many times during the school day would this bell ring at the same time as the Junior School bell?

A village school

Solve problems

Key words

- **ratio** – a comparison of two or more quantities of the same type
- **rate** – a comparison of two different types of measurements

You can solve many practical problems using whole numbers. When you solve money problems, you work with numbers. For example, you work with numbers when planning a budget, taking out a loan or saving money and getting interest. When you calculate distance, speed and time, you also need numbers. For example, if you travel for 3 hours at a speed of 90 km/h, you can work out that the distance travelled is $3 \times 90 = 270$ km.

Ratios and rates

A **ratio** is a useful way of comparing quantities of the same type.

Example

Sam has 8 shells and Thandeka has 12 shells. What is the ratio of Sam's shells to Thandeka's shells?

Answer

The ratio of Sam's shells to Thandeka's shells is 8 to 12. You write this as 8:12.

You usually write a ratio in its simplest form. To do this, divide the numbers by the HCF if there is one. In the example above, the HCF is 4. So dividing the numbers by 4 means that the ratio of Sam's shells to Thandeka's shells can be written as 2:3.

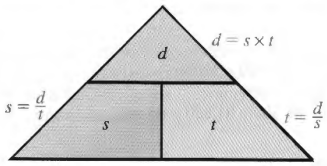
The ratio 2:3 means that for every two shells that Sam has, Thandeka has three shells.

You use ratios when you want to compare quantities of the same type. But when you want to compare two different types of measurement, you can use a **rate**. Some of the most common rates that you will work with in real life are those that involve speed, time and distance.

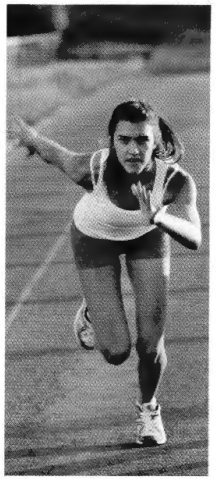
The formulae that we use to calculate speed, time and distance:

$\text{Speed} = \frac{\text{distance}}{\text{time}}$ $\quad \text{Time} = \frac{\text{distance}}{\text{speed}}$ $\quad \text{Distance} = \text{speed} \times \text{time}$

The triangle below can help you to remember these three formulae. In the triangle, d represents distance, s represents speed and t represents time.



You should always specify the units when you write a rate. For example, you can write ten metres per second as 10 m/s and fifteen Rand per kilogram as R15/kg.



Key words

- **VAT** – short for value-added tax, this is tax that is included in the price of goods in a shop
- **hire purchase** – a method of buying goods by making installment payments over time

Discounts and VAT

In South Africa a tax is included in the price of goods in a shop. This is called **Value Added Tax (VAT)** and is 15%. If you buy an article of clothing for R115 this price is made up of R100 for the shopkeeper and R15 which goes to the Government in tax.

Sometimes a shopkeeper offers a discount on goods in the shop. A computer may cost R4 000 but a discount of 20% is given for a few days on a special offer. This means that you will pay R3 200 for the computer as long as the special offer lasts.

Example

1. The price of a laptop before VAT is R3 000. What is the price once VAT has been added?
2. A second hand car dealer offers a discount of 15% on a car priced at R60 000. How much will you have to pay for the car?

Answers

1. 15% of R3 000 = $(\frac{15}{100} \times$ R3 000) = R450. The total cost will be R3 450.
2. 15% of R60 000 = R9 000. You will pay R51 000.

Loans

It is often necessary to borrow money to pay for an expensive purchase, for example a car. You may also need to take out a loan to pay for education. When you borrow money you need to repay the loan with interest. Interest may also be income on an amount saved.

The formula to calculate simple interest is $SI = \frac{prt}{100}$

p = principal (the amount of money borrowed or invested).

r = interest rate used to calculate the interest per year; written as a percentage.

t = time in years for which the principal has been borrowed or invested.

The final amount is the principal plus the interest.

Example

Andile wants to buy a flat screen television set. She takes out a loan of R6 500 @ 12% per annum for 2 years.

a) Calculate the simple interest.
b) What is the final amount she pays?

Answers

a) SI = 6 500 × 12 × 2 ÷ 100 = 1 560
She pays R1 560 interest
b) Final amount = R6 500 + R1 560 = R8 060

Hire purchase

Hire purchase is a method of buying goods through making instalment payments over time. When you need to make a large purchase and cannot afford to pay the full price, you can pay for the article over time and simple interest will be added to the cost of the article. In the meantime you can use the thing you have purchased while paying it off.

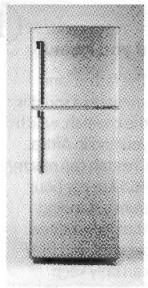
Example

Sipho buys a fridge costing R4 500. He pays 10% deposit and then makes monthly repayments for 2 years to pay for the fridge. The shopkeeper charges him 15% p.a. interest.

a) Calculate the cost of the fridge after the deposit has been paid.
b) Calculate the simple interest charged.
c) How much will he pay every month?

Answers

a) Deposit = 10% of 4 500 = $(\frac{10}{100} \times$ R4 500) = R450
R4 500 – R450 = R4 050
b) 4 050 × 15 × 2 ÷ 100 = 1 215 Interest = R1 215
c) Monthly repayments = (4 050 + 1 215) ÷ 24 = R219,38



Profit and loss

Profit is the difference between the selling price and the cost price of an article. If the selling price is less than the cost price, this is called a loss.

A shopkeeper or retailer purchases his goods for a certain price. This amount is called the cost price or CP. The price that the shopkeeper sells the goods to a customer is called the selling price or SP. Profit (or loss) can be calculated as SP – CP.

Example

A shopkeeper buys goods for R28 000 and sells the goods to his customers at a profit. If his income is R50 400, how much profit does he make?

Answer

CP = 28 000 and SP = 50 400 Profit = 50 400 – 28 000 = 22 400
He makes R22 400 profit

Key words

- **profit** – the difference between the selling price and the cost price of an item
- **exchange rate** – how much it costs to exchange the currency of one country for the currency of another

Exchange rates

Different countries use different currencies. When you travel or do business with other countries, you need to use **exchange rates** to calculate the cost of the other country's currency. These rates change every day, so you need to check with your bank to find out the latest exchange rates when doing a foreign exchange transaction.

Example

Dan is travelling on business and needs to exchange South African rand (called ZAR) into other currencies. He wants to take $1 000, €1 000 and £500. If the rates are R9 to the dollar, R11 to the Euro and R14 to the pound, how much will his foreign currency cost?

Answer

$1 000 = R9 000
€1 000 = R11 000
£500 = R7 000
His currency will cost R27 000.

Did you know?

- The fastest land animal in a sprint is the cheetah, which you find in Africa. Cheetah can reach 96,5 km per hour from a standing start in just three seconds, but they can only keep up this speed for approximately 500 m.
- The slowest mammal in the world is the three-toed sloth of Central and South America. The three-toed sloth has a ground speed of about 0,16 km per hour.

Challenge

- Julia thinks of a number. She increases it in the ratio 4 : 9 and then decreases it in the ratio 2 : 3.
- She then divides her answer by 150 and gets a final answer of 4.
- What was Julia's original number?

Budgets

A budget is a plan designed to show how to spend a certain amount of money, taking different fixed expenses into consideration. Families should draw up a monthly budget to decide how best to spend their income and decide how much money they can save each month. Governments have a budget to decide how much money needs to be spent on Education and other important issues.

EXERCISE 1.6

Use your calculator to check your answers.

1. Write each of these ratios in its simplest form.
   a) 15 : 20 b) 200 : 180 c) 7 : 21 : 35
   d) 102 : 99 : 87 e) 75 m : 125 cm f) 9 min : 600 s
2. Write each of these as a rate. Use the units given in brackets.
   a) Mpho buys four dozen eggs for R48. (R/dozen)
   b) Dan can eat three bananas in 90 seconds. (bananas/min)
   c) A car travels 616 km on 56 litres of petrol. (km/ℓ)
   d) A snail crawls 150 cm in 2 hours. (cm/min)
3. Which is faster: 1 m/s or 1 km/h?
4. Pat is knitting a scarf. On average she can knit 2 cm in 35 minutes.
   a) Express this as a rate in min/cm.
   b) Express this as a rate in mm/min.
   c) How long will it take her to knit a scarf that is 160 cm long?
5. How many times does your heart beat in a day if it beats 70 times per minute?
6. The cost of breakfast cereal is R35 for 1 kg. Calculate how much 25 kg of breakfast cereal will cost.
7. Five litres of paint costs R575. Work out the cost of fifteen litres of paint.
8. If you cycled for 3 hours and travelled 54 km, calculate your average speed.
9. If oil costs $95 per barrel and $1 = R8, how much does a barrel of oil cost in Rand?
10. a) A car travels a distance of 50 km in 30 minutes. What is its average speed?
    b) A car travels at an average speed of 80 km/h for 2 hours. What distance does the car travel?
    c) A car travels a distance of 330 km at an average speed of 110 km/h. How long does it take the car to travel this distance?
11. Share R20 000 among three brothers in the ratio 5 : 3 : 2.
12. Zakele is employed as a technician and earns R5 800 per month. His bills in one month are: Rent R1 500, Electricity R300, Car payment R1 200, Insurance R750, Petrol R400, Groceries R1 000
    Given his current expenses, how much money is available for leisure spending?
13. Calculate the simple interest on a R5 000 loan for 3 years at 9% interest.
14. If a store sells a packet of gum for R5,50, what will the total cost for the gum be if you add the VAT?

Revision

1. Simplify.
   a) $15 + 4 \times 2$ b) $0 \times 50 + 0 \div 18$ c) $125 \div 1 + 32 \times 1$ (3)
2. The radius of the Earth is about $6\frac{1}{2}$ million metres. Write this number in figures. (1)
3. Write this expanded form as a whole number:
   2 000 000 000 + 800 000 000 + 30 000 000 + 8 000 000 + 700 000 + 90 000 + 2 000 + 900 + 40 + 2 (1)
4. Round off 9 882 613 005 to the nearest 1 000 000 (1)
5. Which number is bigger? Fill in < or > in the place of □ in the following:
   a) 2 332 876 □ 1 984 645
   b) 1 805 040 □ 1 850 309 (2)
6. Which number lies half-way between 365 and 1 025? (1)
7. a) Estimate the answer to 21 560 ÷ 98 (1)
   b) Calculate the exact answer to 21 560 ÷ 98 (2)
   c) Work out the difference between your estimation and the answer you calculated. (1)
8. Two numbers multiplied together give a product of 9 600. If one of the numbers is 75, what is the other number? (2)
9. Calculate how many months there are in 21 years. (1)
10. a) Write down the 4th multiple of 8.
    b) Write down two multiples of 8 that are also square numbers. (2)
11. a) Write 72 as the product of its prime factors.
    b) What are the composite factors of 72?
    c) Which factors of 72 are square numbers? (3)
12. List all the prime numbers between 4 and 34. (1)
13. a) Find the prime factors of 280 using a factor tree. (2)
    b) Write 280 as a product of its prime factors. (1)
    c) What are the smallest and greatest factors of 280? (1)
    d) Write down the first three multiples of 280. (1)
14. a) Find the LCM for 28 and 16. (3)
    b) Find the HCF of 210 and 112. (3)
15. Write these ratios in their simplest forms.
    a) 100 minutes:$1\frac{1}{2}$ hours (1)
    b) 15 m:1 050 cm (1)
16. Which ratio is greater, 9:2 or 25:6? (2)
17. If you rode a bicycle for 2 hours and travelled 43 km, calculate your average speed. (1)
18. A car travels a distance of 340 km at an average speed of 85 km/h. How long does it take the car to travel this distance? (2)

Total marks: 40