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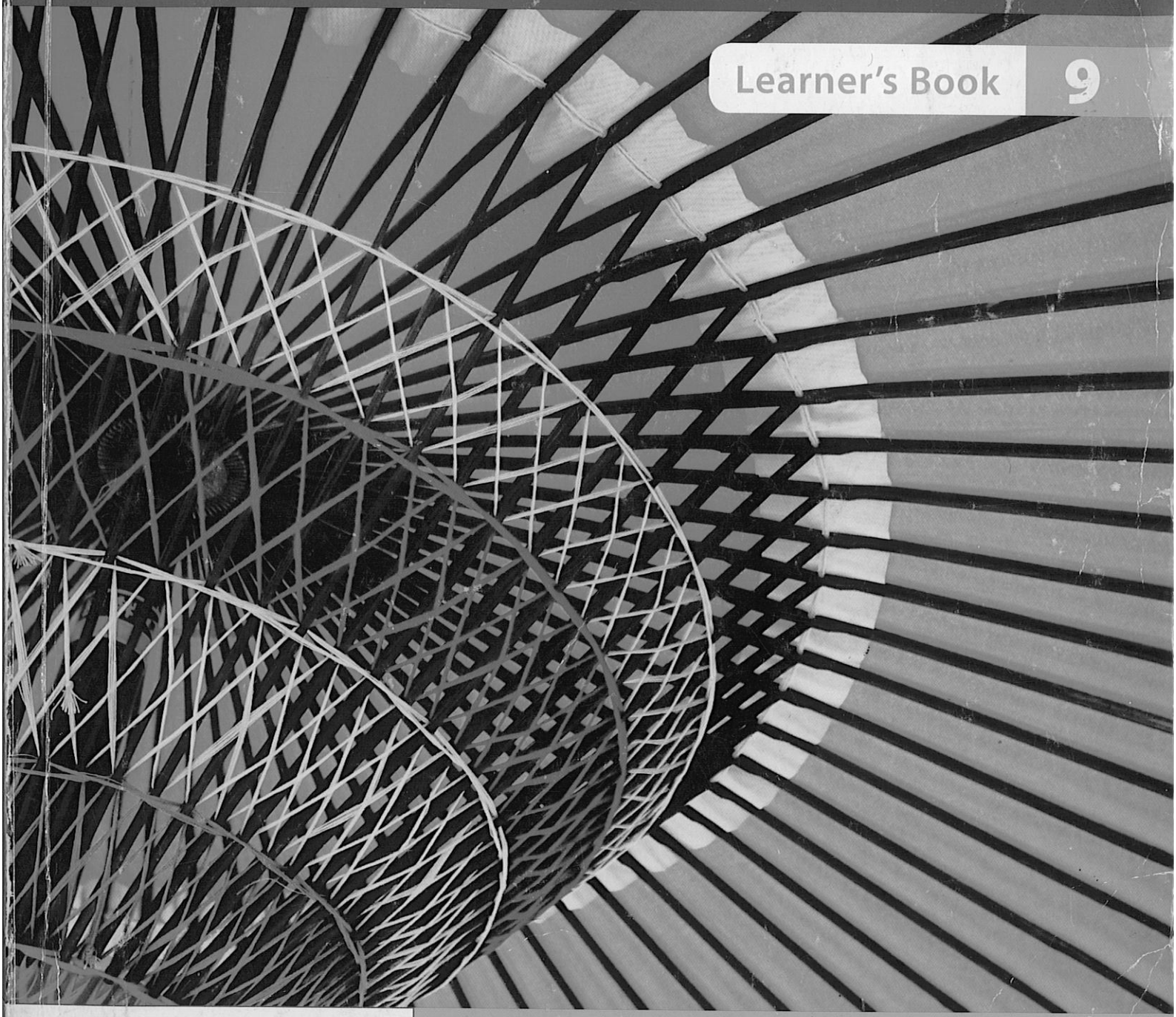

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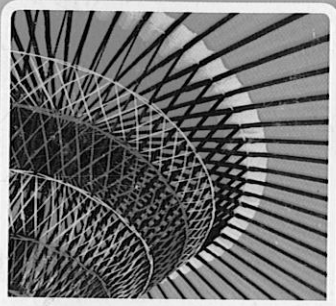

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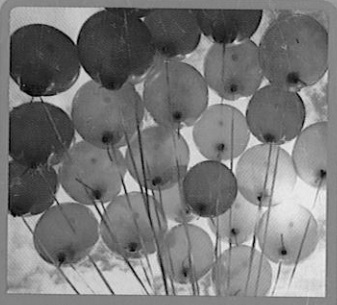

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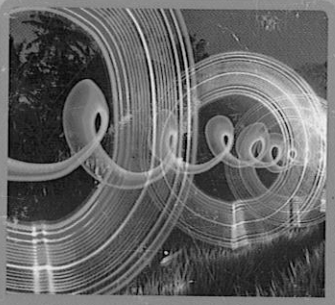

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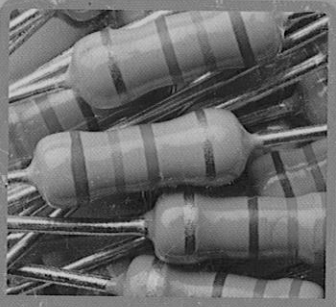

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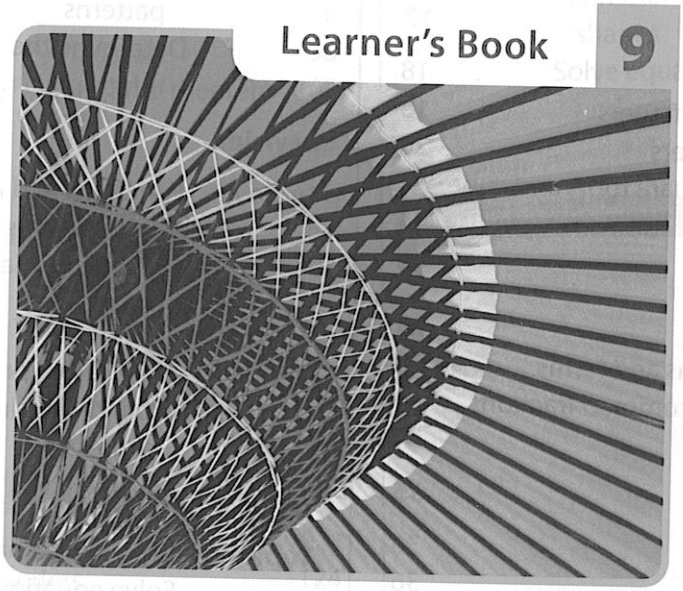
Mathematics

Learner's Book 9



J. Campbell • F. Heany • P. Maritz • B. Rossouw • S. Willers

Consultants: S. Bansilal • V. Frith • M. Langa • H. Lewis

MASKEW MILLER LONGMAN

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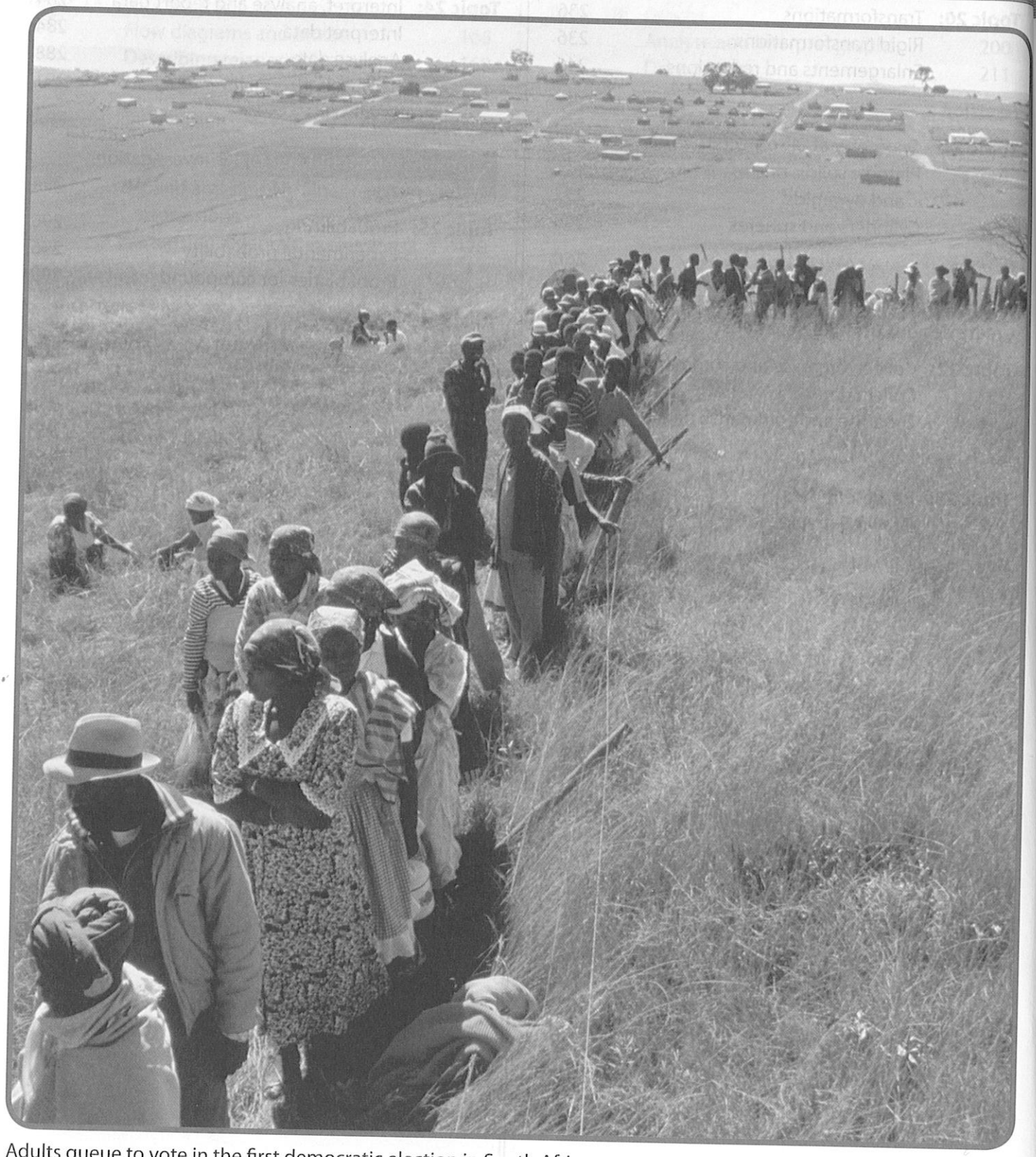
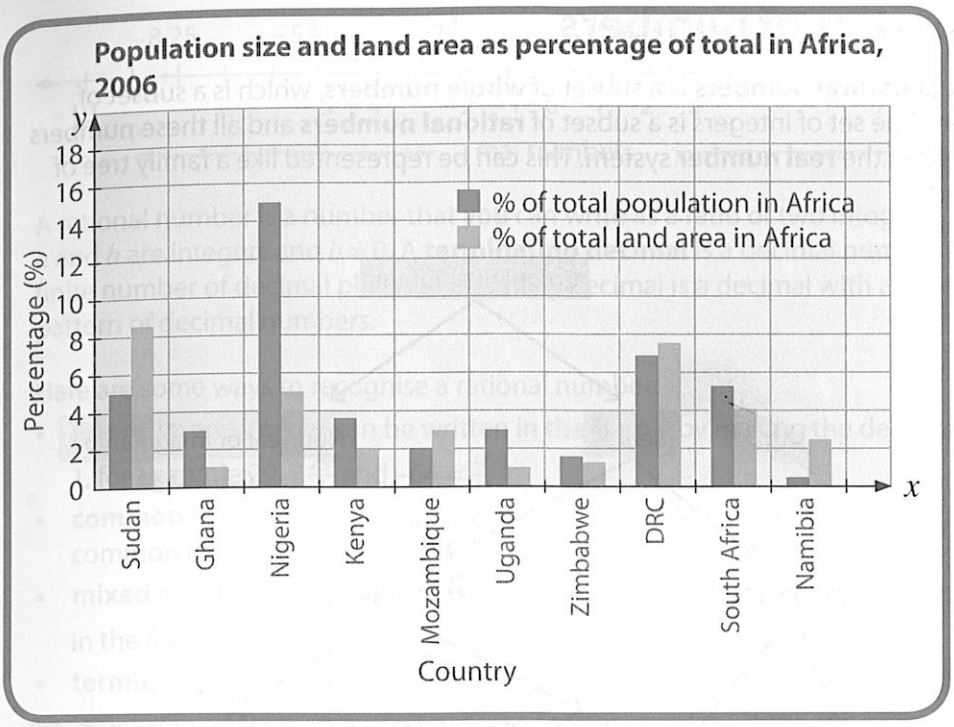


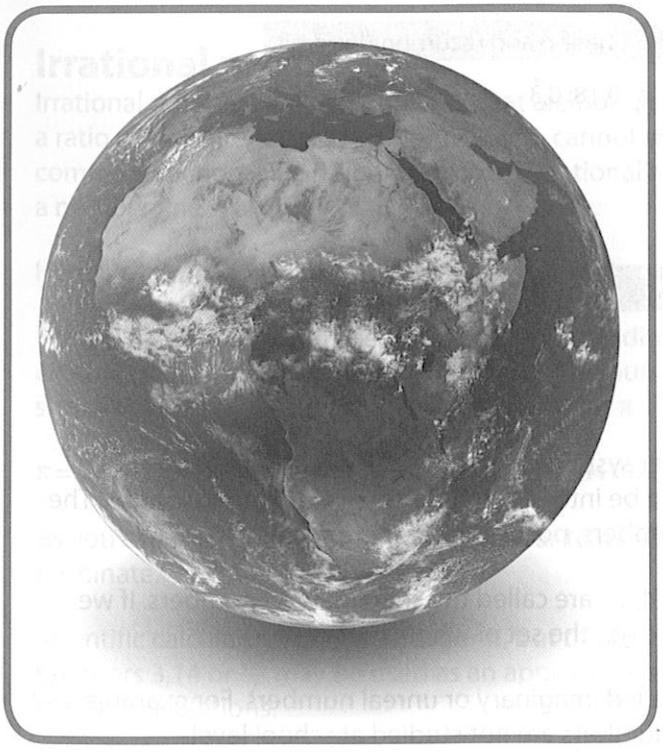
Adults queue to vote in the first democratic election in South Africa.



Starting off

The main picture shows a long queue of adults voting in the first democratic election in South Africa on the 27th April 1994. Millions of South Africans voted for the first time in their lives. The other pictures show the population size and land area as a percentage of the total in Africa, and a world globe.

1. Make a 1 m² paper mat or draw a square with sides 1 m each on the floor. Estimate how many friends can stand next to each other in this square. Check your estimate by standing on the square.



A globe showing the African continent

2. Look at the line of people making their way to the polling booth. Estimate how many people are in the queue to vote.
3. If there were approximately three people in every metre of the queue, determine how long the line of people was if there were 1 500 people in this queue.
4. In 2012, there were 1 032 533 000 people in Africa. If the population of South Africa was 50 738 000, what percentage of the people in Africa lived in South Africa?
5. The world population passed the 6 billion mark just before the end of the 20th century. If the average rate of population growth is 3 people per second, how long will it take for the population to double?

Content covered in Term 1

Topic 1: Whole numbers, **Topic 2:** Integers, **Topic 3:** Common fractions, **Topic 4:** Decimal fractions, **Topic 5:** Exponents, **Formal Assessment Exemplar:** Assignment – Exponents in science, **Topic 6:** Numeric and geometric patterns, **Topic 7:** Functions and relationships, **Topic 8:** Algebraic expressions, **Topic 9:** Algebraic equations, **Formal Assessment Exemplar:** Test

Topic 1

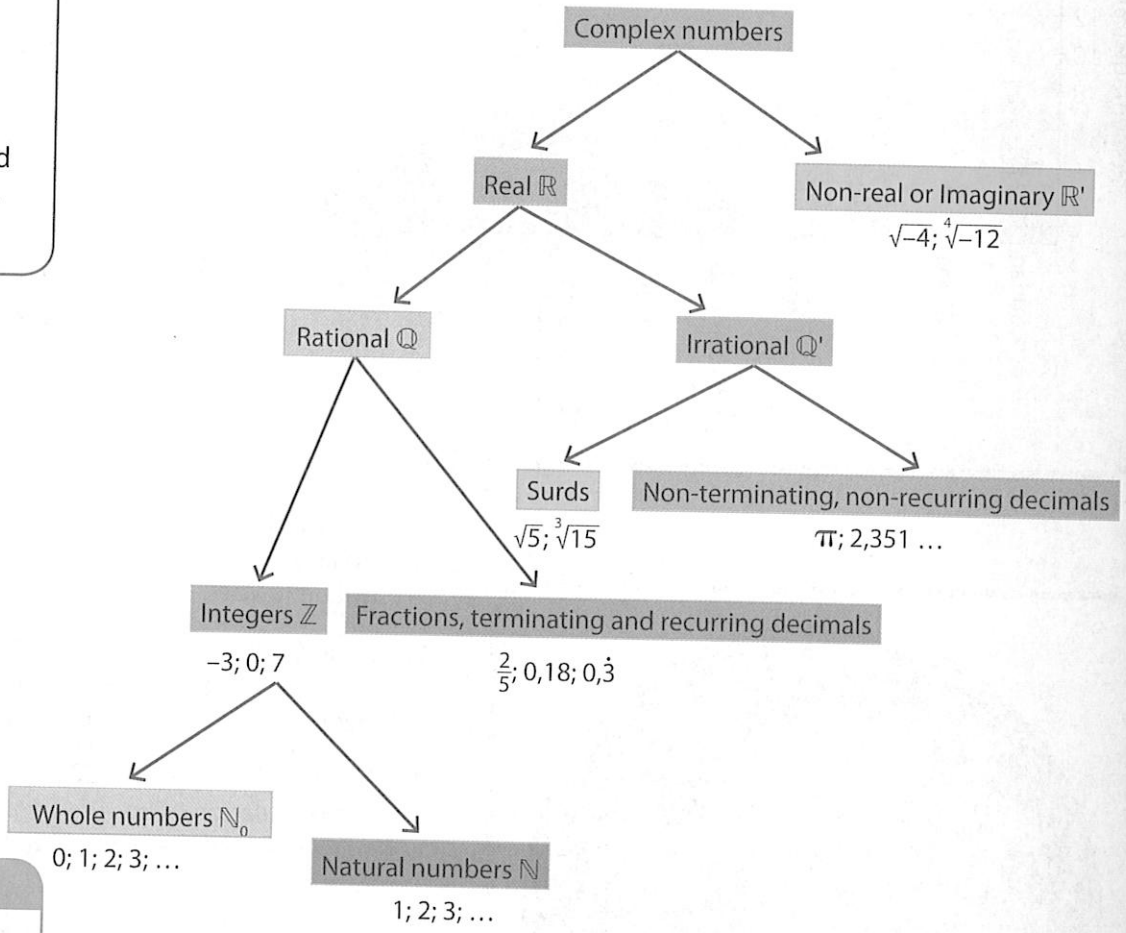
Whole numbers

Maths ideas

- Recognise the properties of numbers.
- Calculate with whole numbers.
- Estimate, round off and use a calculator.
- Find the LCM and HCF of numbers.
- Solve problems.

Properties of numbers

The set of **natural numbers** is a subset of **whole numbers**, which is a subset of **integers**. The set of integers is a subset of **rational numbers** and all these numbers form part of the **real number** system. This can be represented like a family tree of numbers.



Key words

- **natural numbers** – whole numbers greater than or equal to 1: {1; 2; 3; 4; …} and are represented by the symbol ℕ
- **whole numbers** – the numbers 0; 1; 2; 3; 4; … which are represented by the symbol $\mathbb{N}_0$

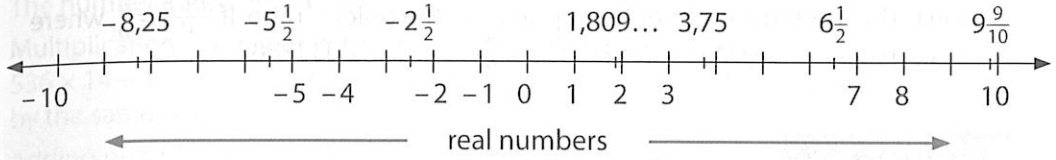
You can see that the real number system consists of all rational and **irrational numbers**. Rational numbers can be integers, fractions or **recurring decimals**. The integers include all negative numbers, positive numbers and zero.

All the positive numbers 1; 2; 3; 4; … are called the set of natural numbers. If we add 0 to the natural numbers we get the set of whole numbers.

Numbers that are not real are called imaginary or unreal numbers. For example $\sqrt{-2}$ has no real solution and these numbers are not studied at school level.

Rational numbers

In Grade 8 you studied whole numbers and integers. All the other numbers on a number line that make up all the real numbers are either rational or irrational. Here is a number line showing a few rational numbers and an irrational number.



A rational number is a number that you can write as a ratio of two integers $\frac{a}{b}$ where a and b are integers and $b \neq 0$. A **terminating decimal** is a decimal number with a finite number of decimal places. A recurring decimal is a decimal with a repeating pattern of decimal numbers.

Here are some ways to recognise a rational number:

- **integers:** any integer can be written in the form $\frac{a}{b}$ by making the denominator 1, for example $29 = \frac{29}{1}$ and $-64 = \frac{-64}{1}$
- **common fractions:** all common fractions are written in the form $\frac{a}{b}$ so all common fractions are rational
- **mixed numbers:** all mixed numbers may be re-written as an improper fraction in the form $\frac{a}{b}$, for example $2\frac{19}{20} = \frac{59}{20}$
- **terminating decimals:** you can write any decimal that terminates as a fraction with a denominator that is a power of 10, for example $6{,}2365 = 6\frac{2\,365}{10\,000} = \frac{12\,473}{2\,000}$
- **recurring decimals:** a recurring decimal can be written as a fraction, for example $0{,}3333333\ldots = \frac{1}{3}$.

Irrational numbers

Irrational numbers are real numbers that are not rational. They cannot be written as a ratio of two integers. In other words, you cannot write an irrational number as a common fraction. Like rational numbers, irrational numbers can also be shown on a number line.

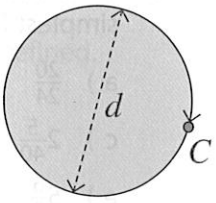
Here are some famous irrational numbers used by mathematicians:

π (pi) is the irrational number that you get if you divide the circumference of any circle by its diameter. The number below shows the first 50 decimal places in the value of π.

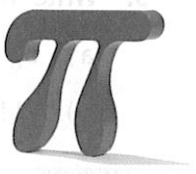
$\pi = 3{,}14159265358979323846264338327950288419716939937510\ldots$

As you can see, the decimals do not appear to make a pattern or terminate.

Scientific calculators have a value for π, but the rational numbers 3,14 or $\frac{22}{7}$ may be used as an approximate value for π in calculations.



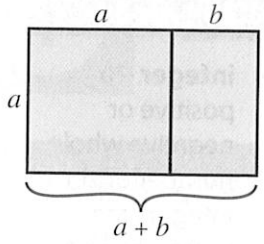
$C = \pi d$



The symbol for pi

Key words

- **integer** – a positive or negative whole number or zero; integers are represented by the symbol $\mathbb{Z}$
- **rational numbers** – can be written in the form $\frac{a}{b}$ where a and b are both integers and $b \neq 0$; rational numbers are represented by the symbol $\mathbb{Q}$
- **real numbers** – all the rational and irrational numbers; real numbers are represented by the symbol $\mathbb{R}$
- **irrational numbers** – real numbers that cannot be written in the form of a rational number
- **recurring decimal** – a decimal with a repeating pattern of decimal numbers, for example $0{,}1616161616\ldots$ or $0{,}\dot{1}\dot{6}$
- **terminating decimal** – a decimal number with a finite number of decimal places, for example 7,125 or 0,9752



$\sqrt{2}$, the square root of 2, is an irrational number. It is approximately equal to 1,41421356...

You will use this square root in later grades when you study Trigonometry.

φ (phi) is the irrational number that appears in the golden ratio. If $\frac{a+b}{a} = \frac{a}{b}$, where $a > b$, then a and b are in the golden ratio. $\frac{a}{b} = \varphi = 1{,}6180339887...$

Did you know?

$\sqrt{2}$ is sometimes called Pythagoras' constant. According to legend, one of the Pythagoreans, Hippasus, discovered during a sea voyage that $\sqrt{2}$ was not a rational number. When he told his companions, they threw him overboard because they believed all numbers should be rational.

EXERCISE 1.1

1. Decide whether the following numbers are rational or irrational numbers:

$\sqrt{25}$; $-65{,}2$; $-\sqrt{81}$; $\frac{22}{7}$; $3{,}14$; π ; $3\sqrt{15}$; $\sqrt[3]{-27}$

2. Given the list of numbers $-7{,}7$; 5 ; $\sqrt{-6}$; 8; $\sqrt{35}$; 36 ; $\frac{71}{3}$; write down:

a) a prime number
b) an irrational number
c) a factor of 32
d) a square number
e) an unreal number.

3. Classify each number below as an integer, fraction, mixed number or decimal number:

a) $\frac{125}{500}$
b) $-0{,}67$
c) $5\,416$
d) $6\frac{2}{5}$
e) $\frac{18}{45}$
f) $0{,}375$
g) $1{,}397$
h) $-5\frac{7}{9}$
i) $6\,903\,651$
j) $\frac{27}{918}$
k) $-436{,}029$
l) $12\frac{3}{4}$

4. Write each of the rational numbers below as a common fraction in its simplest form (without the use of a calculator):

a) $\frac{20}{24}$
b) $-\frac{48}{80}$
c) $2\frac{5}{40}$
d) $-7\frac{25}{200}$
e) $5\frac{6}{8}$
f) $\frac{85}{170}$

5. Write the following mixed numbers as improper fractions.

a) $6\frac{4}{9}$
b) $-4\frac{5}{6}$
c) $-11\frac{2}{3}$
d) $13\frac{7}{15}$

Challenge

Calculate the square roots of the first 10 positive integers. Draw a number line that shows their approximate positions. Use your calculator where needed.

Calculations with numbers

In Mathematics, addition, subtraction, multiplication and division are called operations.

Addition and subtraction are inverse operations, for example 236 + 18 – 18 = 236. The number 236 stays the same if you add and subtract the same number.

Multiplication and division are inverse operations of each other, for example 536 × 14 ÷ 14 = 536. The number 536 stays the same if you multiply and divide by the same number.

Adding numbers is called finding the **sum** and subtracting numbers is called finding the **difference**. Multiplying numbers is called finding the **product**. A **quotient** is the result of dividing numbers.

Key words

- **sum** – the result of addition
- **difference** – the result of subtraction
- **product** – the result of multiplication
- **quotient** – the result of division

Properties of rational numbers

Rational numbers, like whole numbers, have various calculation properties:

- For any two rational numbers A and B: $A + B = B + A$ and $AB = BA$
  When you add or multiply numbers, the order of the numbers does not matter, for example: 3,25 + 6,18 = 6,18 + 3,25 and $\frac{1}{4} \times \frac{3}{5} = \frac{3}{5} \times \frac{1}{4}$
  This is called the commutative property of addition and multiplication.

- For any three rational numbers A, B and C: $(A + B) + C = A + (B + C)$ and $(A \times B) \times C = A \times (B \times C)$
  For example: (1,2 + 3,4) + 5,6 = 1,2 + (3,4 + 5,6) and (2,4 × 1,3) × 3,6 = 2,4 × (1,3 × 3,6)
  This is called the associative property of addition and multiplication.

- For any rational numbers A, B and C: $A(B + C) = AB + AC$ and $A(B - C) = AB - AC$
  For example: 2,1(3,6 + 2,9) = 2,1(3,6) + 2,1(2,9) and 2,1(3,6 – 2,9) = 2,1(3,6) – 2,1(2,9)
  You can simplify inside the brackets where possible: 2,1(3,6 + 2,9) = 2,1(6,5)
  This is called the distributive property of multiplication.

There are other useful properties of rational numbers. We call them the inverse and identity properties.

The inverse properties of rational numbers states that $A + (-A) = 0$ and $A \times \frac{1}{A} = 1$

The identity properties of rational numbers states that $A + 0 = A$ and $A \times 1 = A$

Multiplying any number by zero gives a result of zero: 1,94 × 0 = 0

You may not divide by zero as the answer is undefined. 1,94 ÷ 0 = undefined.

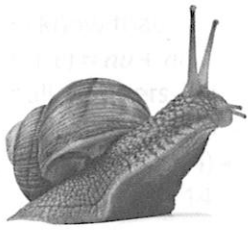
The order of operations is important in Mathematics.

Simplify calculations inside brackets first where possible, for example:
2,5(16,5 + 8,5) = 2,5(25) = 62,5

Multiply and divide before adding and subtracting, for example:
17 + 18 × 23 = 17 + 414 = 431, but (17 + 18) × 23 = 35 × 23 = 805

Did you know?

A garden snail named Archie won the World Snail Racing Championship held in England in 1995. He covered 13 inches in 2 minutes and 20 seconds. It is still the world record.



Example

Simplify the following:

a) $2(\frac{1}{2} + 3)$ b) $\frac{1}{4} \times a \times 4$ c) $3\,125 \times 0 + (\frac{3}{8} \times \frac{8}{3} - 1)$

Answers

a) $2(3\frac{1}{2}) = 2(\frac{7}{2}) = 7$ b) $\frac{1}{4} \times 4 \times a = 1 \times a = a$ c) $0 + (1 - 1) = 0 + 0 = 0$

Did you know?

Many mobile phones have a built-in calculator. You could have completed Exercise 1.2 by using a cellphone to help you with the calculations.

Using a calculator

SHARP EL-W535HT: Use the change button to write a number as a mixed fraction, common fraction or decimal.

CASIO FX 82: Use the SD button to change from decimal to fraction form.

When you calculate with irrational numbers, your answer will always be an approximation.

Step 1: Choose which rational approximation of the irrational number you will use, for example if your calculator does not have a π button, you might choose to use 3,14 or $\frac{22}{7}$

Step 2: Perform the calculation.
Step 3: Round off the answer to a suitable degree of accuracy.
Step 4: Check that your answer seems reasonable.

Addition

You can add numbers horizontally or in columns. You may use a calculator to check the answers.

Example

Find the sum of 384 + 527 + 472

Answer

Adding horizontally:

(300 + 500 + 400) + (80 + 20 + 70) + (4 + 7 + 2)
= 1 200 + 170 + 13
= 1 200 + 100 + 70 + 10 + 3
= 1 300 + 80 + 3
= 1 383

Using columns:

	H	T	U	
	$^{1}3$	$^{1}8$	4	← In the units column 4 + 7 + 2 = 13. Write 3 in the units column and add 1 to the tens column
	5	2	7	← In the tens column 8 + 2 + 7 + 1 = 18. Write 8 in the tens column and add 1 to the hundreds column
+	4	7	2	← In the hundreds column 3 + 5 + 4 + 1 = 13. Write 3 in the hundreds column and 1 in the thousands column
	1	3	8	3

Subtraction

Write the numbers directly underneath each other in columns and then subtract them. Use your calculator to check the answer.

Example

Find the difference between 783 and 529

Answer

	H	T	U	
	7	$^{7}8$	$^{1}3$	← You need to take a ten from the tens column. Then subtract 9 from 13
−	5	2	9	← 8 tens has been reduced to 7 tens
	2	5	4	

Multiplication

When you multiply larger numbers together, use **long multiplication.** By using this method, you break a difficult product into the sum of simple products.

Example

Calculate 5 605 × 25.

Answer

5 605 × 25 = 5 605 × (20 + 5)
= (5 605 × 20) + (5 605 × 5) ← use the distributive property
= 112 100 + 28 025
= 140 125

In columns, this looks like:

	5 605	
×	25	
	28 025	← 5 605 × 5 (multiply by units)
+	112 100	← 5 605 × 20 (multiply by tens)
	140 125	← Add the two products together

Division

When you divide large numbers you can use a method called **long division** using columns.

Example

Divide 134 880 by 240

Answer

562	← This number is the answer
240)134880	
1200	← 240 × 5 = 1 200
1488	← Subtract 1 200 from 1 348 and bring down the 8
1440	← 240 × 6 = 1 440
480	← Subtract 1 440 from 1 488 and bring down the 0
480	← 240 × 2 = 480
0	← Subtract 480 from 480

Challenge

You know that $a(b + c) = ab + ac$ for all numbers. For example, 14 = 2 × 7 = 2 × (4 + 3) = (2 × 4) + (2 × 3) = 8 + 6 = 14.

- Check this property for negative numbers. Fill in the spaces: −2 = −2 × 1 = −2 × (4 + (−3)) = (−2 × 4) + (−2 × (−3)) = ... + ... = ...
- Make up your own example using negative numbers. Start with a positive number and represent it as a product of two negative numbers.

When doing any mathematical calculations, you should always estimate what you think the answer should be and check whether your answer makes sense.

EXERCISE 1.2

1. Simplify the following:

a) $6(\frac{1}{2}+\frac{1}{3})$ b) $\frac{1}{2}(18+36)$

c) $12(13-8)-11(22-17)$ d) $1{,}5\times 3+1{,}5\times 4$

e) $\frac{1}{8}\times 2\times 8$ f) $\frac{3}{4}\times\frac{4}{3}\times 12$

2. Calculate:

a) $345\,692\times 0$ b) $671{,}324\times 1$

c) $125\times 0+(\frac{3}{5}\times\frac{5}{3})$ d) $(0\times 5{,}17)+(0\div 6{,}25)$

e) $(\sqrt{25}\div 1)+(\sqrt{26}\div 0)$ f) $7{,}125+0-9{,}287\times 0$

3. Simplify the following:

a) $0{,}8\times 7+4{,}5\times 3$ b) $175+18\times 2$

c) $120\times 3+110\times 4$ d) $\frac{1}{5}\times\frac{5}{3}+\frac{7}{3}\times\frac{2}{7}$

4. Solve:

a) $150\times(30+120)$ b) $180+(400+240)$

c) $72(400-300)$ d) $96+(-96)+32\div 32-25\times\frac{1}{25}$

5. Say whether the following statements are true or false:

a) The sum of two rational numbers is always a rational number.

b) The difference between two rational numbers is always a rational number.

c) The product of two rational numbers is always a rational number.

d) The quotient of two rational numbers is always a rational number.

6. Calculate:

a) 45 231 + 7 050

b) 140 006 + 23 987

c) 8 950 – 429

d) 18 532 – 6 732

7. Use long multiplication to multiply the following and check the answers on your calculator:

a) $2\,189\times 21$

b) $6\,034\times 36$

c) $50\,201\times 45$

d) $6\,891\times 720$

8. Use long division to divide the following and check the answers on your calculator:

a) $8\,700\div 60$

b) $34\,719\div 71$

c) $208\,640\div 160$

d) $102\,024\div 156$

Multiples and factors

We find the **factors** of a whole number by finding all the numbers that divide exactly into the original number. A number is **divisible** by another if, after dividing, there is no remainder.

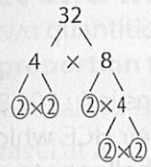
All numbers may be written as a product of their **prime** factors. You can use a factor tree to write a number as the product of its prime factors.

Example

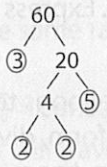
a) Write 32 as a product of its prime factors
b) Write 60 as a product of its prime factors

Answer

a) $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$



b) $60 = 2^2 \times 3 \times 5$



Highest common factor (HCF)

The largest common factor of two or more numbers is called the **highest common factor** or **HCF**.

Lowest common multiple (LCM)

A multiple is the product of two natural numbers. The multiples of any number are simply the times table for that number, for example the multiples of 12 are 12, 24, 36, 48, … You can find the **lowest common multiple (LCM)** of two or more numbers. Write each number as a product of its prime factors. The LCM must contain all the factors from each number without duplication. Any common factors are only used once.

Example

Find the HCF and LCM of 32 and 60 using prime factors

Answer

HCF of 32 and 60 = $2 \times 2 = 4$

LCM of 32 and 60 = $2^5 \times 3 \times 5 = 480$

EXERCISE 1.3

1. List the factors of 27 and 36 and then find the HCF.
2. a) Write the first five multiples of 6 and 8.
   b) What is the LCM of 6 and 8?
3. a) List the prime numbers up to 40.
   b) Is 57 a prime number? Explain.
4. Find the HCF and LCM for these sets of numbers using prime factors:
   a) 40 and 56 b) 60 and 150 c) 36 and 84.
5. Aysha has 136 buttons and eight containers. Can she divide the buttons equally between the containers?

Key words

factor – a number that divides exactly into a whole number with no remainder

divisible – a number is divisible if, after dividing, there is no remainder

prime numbers – numbers that have only two factors, the number itself and 1

highest common factor (HCF) – the highest factor that is common to two or more numbers

multiple – the product of two natural numbers

lowest common multiple (LCM) – the lowest multiple that is common to two or more numbers

Did you know?

The HCF is useful when simplifying fractions; and the LCM is useful when adding and subtracting fractions.

Key words

- **ratio** – a comparison of two or more quantities of the same type
- **rate** – a comparison of two different types of measurements

Solve problems

Ratios and rates

A **ratio** is used to compare two or more quantities of the same type. It is written in the form $a : b$ where a and b are integers.

Example

1. There are 4 green, 10 blue and 8 red balloons in a packet. Express this as a ratio in its simplest form.
2. Baking powder can be made by mixing $\frac{1}{4}$ T of bicarbonate of soda, $\frac{1}{2}$ T cream of tartar and $\frac{1}{4}$ T of cornflour together (1 T = 1 tablespoon). Express this as a ratio.

Answers

1. The ratio of green balloons to blue balloons to red balloons is 4 : 10 : 8
   To find the simplest form, divide all three quantities by their HCF, which is 2. In its simplest form, the ratio of green balloons to blue balloons to red balloons is 2 : 5 : 4
2. Use integers to express a ratio. Multiply all the quantities by the lowest common denominator to get rid of the fractions. Multiply all the fractions by 4 and the ratio of bicarbonate of soda to cream of tartar to cornflour is 1 : 2 : 1

A **rate** is a special kind of ratio that expresses the relationship between two different types of measures such as kilometres and hours, or price and volume. It is important to use the correct units when you write down a rate. You can write a rate by using the word 'per' or using a forward slash (/), for example R17,95 per kilogram or R17,95/kg.

Example

1. A dripping tap wastes 50 ml of water per minute. How much water does it waste in a day?
2. Determine which loaf is the best value for money:

   Loaf A, which costs R9,65 for 700 g; Loaf B, which costs R10,80 for 800 g or Loaf C, which costs R11,90 for 900 g?

Answers

1. Calculate the number of minutes in a day.
   60 (minutes in an hour) × 24 (hours in a day) = 1 440 minutes in a day
   Water wasted in one day = 50 ml × 1 440 minutes
   = 72 000 ml = 72 ℓ
2. Loaf A: R9,65 for 700 g is equivalent to (R9,65 ÷ 7) for 100 g = R1,38 per 100 g
   Loaf B: R10,80 for 800 g is equivalent to (R10,80 ÷ 8) for 100 g = R1,35 per 100 g
   Loaf C: R11,90 for 900 g is equivalent to (R11,90 ÷ 9) for 100 g = R1,32 per 100 g
   Loaf C is the best value for money as the price per 100 g is the lowest.

Speed, distance and time

Some of the most common rates that you will work with in real life involve speed, time and distance.

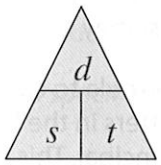
The formulae that we use to calculate distance, speed and time are:

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{Distance} = \text{speed} \times \text{time}$$

$$\text{Time} = \frac{\text{distance}}{\text{speed}}$$

In the triangle, d represents distance, s represents speed and t represents time.



This triangle can help you to remember these formulae.

Direct and indirect proportion

When two quantities increase or decrease by the same ratio, we say that they are in **direct proportion** to each other.

Mathematically, x is directly proportional to y if $\frac{x}{y}$ = constant.

If you travel at a constant speed of 90 km/h, you will travel 180 kilometres in 2 hours; but if you travel for 4 hours at the same speed you will travel 360 km.

The fractions $\frac{180}{2}$ and $\frac{360}{4}$ are equivalent and these ratios are in direct proportion.

Two quantities x and y are inversely proportional or **indirectly proportional** to each other if, as the value of x increases, the value of y decreases and the product of the values is constant: $x \times y$ = a constant. For example a farmer hires 12 men to construct a building. The job is completed in 18 days. If he had hired 24 men, the job would have been completed in half the time. If he had only hired 6 men, the job would have taken twice as long to complete.

$12 \times 18 = 216$ and $24 \times 9 = 216$ and $6 \times 36 = 216$.

As the number of men employed increases, the time taken decreases proportionally.

As the number of men employed decreases, the time taken increases proportionally.

The total number of hours needed to get the job done remains constant (216 hours).

Discounts and VAT

In South Africa a tax is included in the price of goods in a shop. This is called **Value Added Tax (VAT)** and is 14%. If you buy an article of clothing for R228, this price is made up of R200 for the shopkeeper and R28, which goes to the Government in tax.

Sometimes a shopkeeper offers a **discount** on goods in the shop. A computer may cost R5 400 but a discount of 15% is given for a few days on a special offer. This means that you will pay R4 590 for the computer as long as the special offer lasts.

Example

1. The price of a laptop before VAT is R4 385. What is the price once VAT has been added?
2. A secondhand car dealer offers a discount of 18% on a car priced at R115 000. How much will someone have to pay for the car?

Answers

1. 14% of R4 385 = R613.90. The total cost will be R4 998,90.
2. 18% of R115 000 = R20 700. They will pay R94 300.

Key words

- **direct proportion** – x and y are directly proportional when there is a constant multiplier. The value of x increases (or decreases) in the same proportion as the value of y increases (or decreases)
- **indirect proportion** – x and y are indirectly proportional when an increase in x matches a decrease in y or a decrease in x matches an increase in y
- **VAT** – a value added tax on goods and services
- **discount** – a reduction in the original price of something

Loans

It is often necessary to borrow money to pay for an expensive purchase, for example a car. You may also need to take out a loan to pay for education. When you borrow money you need to repay the loan with interest.

Interest may be simple or compounded.

The formula to calculate simple interest is $SI = \frac{Prt}{100}$
The letters in the formula are:
P = principal. This is the amount of money borrowed or invested
r = interest rate. This is the rate used to calculate the interest per year and it is written as a percentage.
t = time in years. This is the number of years for which the principal has been borrowed or invested.

The final amount is the total money and is equal to the principal plus the interest.

When interest is compounded, interest is paid on interest. For example if R1 000 is deposited in a bank earning 9% per annum compounded annually, the interest earned at the end of the first year is R90. The total amount in the bank is now R1 090. At the end of the second year, 9% is calculated on the new total.
R1 090 $\times \frac{9}{100}$ = R98,10. Each year, the interest is calculated on the new balance.

Example

Sizakele wants to buy a new television. He takes out a loan of R8 500 @12% per annum for 2 years.

a) Calculate the simple interest.
b) What is the total amount he pays?

Answers

a) SI = 8 500 × 12 × 2 ÷ 100 = 2 040
He pays R2 040 interest.
b) Final amount = R8 500 + R2 040 = R10 540

Profit and loss

Profit is the difference between the selling price and the cost price of an article. If the selling price is less than the cost price, this is called a loss.

A shopkeeper or retailer purchases goods at a certain price. This amount is called the cost price or CP. The price that the shopkeeper sells the goods to a customer is called the selling price or SP.

Profit (or loss) can be calculated as SP – CP.

Example

A shopkeeper buys goods for R28 000 and sells the goods to his customers at a profit. If his income is R50 400, how much profit does he make?

Answer

CP = 28 000 and SP = 50 400
Profit = 50 400 – 28 000 = 22 400
He makes R22 400 profit.

Budgets, accounts, rentals and commission

A **budget** is a plan designed to show how to spend a certain amount of money taking different fixed expenses into consideration. Families should draw up a monthly budget to decide how best to spend the income and decide how much money they can save each month. Governments draw up a budget to decide how much money needs to be spent on Education and other important issues.

Rental is the amount of money that you pay to use something for a specified period of time, for example, rent a car or pay rental on a property.

A bank **account** is an arrangement in which a bank keeps money for you and you can access the account to deposit or withdraw funds. You may also open an account at a shop. This is a facility that lets you make purchases and pay them off over a fixed period of time. **Commission** is an amount of money that you are paid every time you sell something.

Key words

- **budget** – a plan to show how to spend a certain amount of money
- **rental** – the amount of money that you pay to use something for a specified period of time
- **account** – a business arrangement between a bank and a client or a shop and a customer
- **commission** – payment for goods sold

Example

1. Beatrice has an account at a large department store. Her account balance is R3 924 and she needs to pay it off in equal instalments over 6 months. Calculate the cost of each repayment.
2. Temoso sells T-shirts at a market and earns 15% commission on each T-shirt he sells. How many T-shirts must he sell to make R360 if the cost of each T-shirt is R120?

Answers

1. $3\,924 \div 6 = R654$
   She needs to repay R654 every month.
2. 15% of R120 = R18
   $360 \div 18 = 20$
   He must sell 20 T-shirts.

EXERCISE 1.4

Write all ratios in their simplest form.

1. There are three children in the Skosana family. Each week the eldest earns R270, the middle child earns R180 and the youngest earns R90. Express these amounts as a ratio.
2. To make a certain colour of household paint, you need to mix $\frac{3}{4}\,\ell$ of white paint with 15 ml of red paint and 10 ml of yellow paint. Express the paint mixture as a ratio. If you need a total volume of 31 litres of this paint, how much of each colour must you mix?
3. On Wednesday evening Sonto spent $\frac{1}{2}$ hour doing homework, 45 minutes doing housework, 1 hour visiting friends and $1\frac{1}{2}$ hours watching television. Write this information as a ratio.
4. In a country there are 48 000 people in the defence services. They work in the army, navy and air force in the ratio 5 : 2 : 3. How many people are in each service?

Did you know?
The human heart beats at an average rate of 72 beats per minute.

Challenge

Estimate the number of times your heart has beaten since you were born.

5. To make a certain drink, you need to mix 200 ml of concentrate with 1,2 ℓ of water. Determine the ratio of concentrate to water. How much concentrate will you need if you have to make up a total of 14 litres of this drink?

6. A car travelling at a constant speed travels 90 km in 50 minutes. How far, travelling at the same constant speed, will the car travel in 2 hours and 25 minutes?

7. Calculate the simple interest on R1 200 at 7% p.a. for 3 years using the formula $SI = \frac{Prt}{100}$
(P = principal amount, t = time (number of years), r = rate of interest and SI = simple interest).

8. How long will it take for R6 000 invested at 6% per annum simple interest to grow to R8 520? (Use the formula $SI = \frac{Prt}{100}$)

9. Temoso borrowed R1 000 from the bank for 3 years at 8% compound interest. Without using a formula, calculate how much Temoso owes the bank at the end of three years.

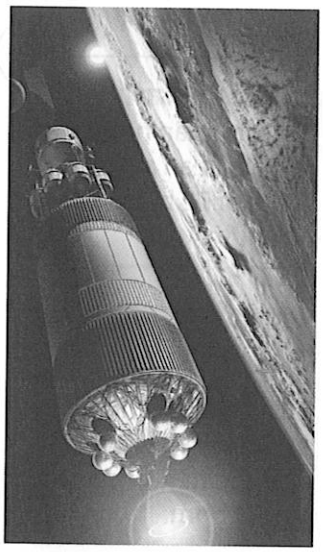
10. Tom pays R1 250 rent every month. What is his annual rent? If his rent is increased by 8,5% at the end of the year, what will his increased monthly rent be?

11. It takes 96 hours for three grape pickers to clear a vineyard. Complete the table below:

pickers	3	6	12	18	24
hours	96				

12. It takes three men six hours to repair a road. How long would it have taken two men?

13. Zakele needs to draw up a budget of his monthly expenses. He earns R6 500 per month. His bills in one month are: Rent R1 650, Electricity R300, Car payment R1 200, Insurance R750, Petrol R400, Account repayments R500 and Groceries R1 200. Given his current expenses, how much money is available for saving?



14. a) Calculate the simple interest on a R5 000 loan for 3 years at 8.5% p.a. interest.
    b) Calculate the compound interest earned on R15 000 deposited into a bank for 2 years @12% p.a. interest.

15. Value Added Tax (VAT) of 14% is added to all goods sold in South Africa. If Teago paid R5,45 for a soda drink at the counter, what amount of the price he paid goes to VAT? Round off your answer to the nearest cent.

Challenge

A spacecraft travels 360 000 km to the Moon at an average speed of 1 200 m/s. How long does the journey take in days, hours and minutes?

Revision

1. Choose the correct term from the box below to complete the sentences that follow:

real numbers	square root
integers	terminating decimal
square numbers	irrational number

a) Rational and irrational numbers together form the set of … (1)

b) The numbers –129; –66; 0; 43 and 9 876 are all classified as … (1)

c) The … of 64 is 8. (1)

d) π or pi is an … that describes the ratio of the circumference of a circle to its diameter. (1)

e) 9; 16; 121 and 144 are … (1)

2. List the rational numbers among the following:

$\frac{29}{84}$; 0,165 ; π ; $-\frac{18}{7}$; 0,44444444 … ; –7,375 ; $\sqrt{42}$; $6\frac{1}{2}$; $0,\dot{2}\dot{3}\dot{4}$ (1)

3. Use the properties of numbers to simplify the following:

a) $2(\frac{3}{2}+0,5)+\frac{1}{2}$ (1)

b) $1\,000\times 5\times\frac{2}{5}+1\times 0$ (1)

c) $\frac{5}{6}-0\times\frac{6}{5}$ (1)

d) $6\times 3\times\frac{1}{18}+9+0$ (1)

4. List the irrational numbers from the following:

$\frac{22}{7}$; –99,191919… ; $\sqrt{54}$; $0\dot{6}$; π ; $\sqrt[3]{64}$; 3,142 (1)

5. a) Estimate the answer to: 422 801,15 ÷ 101 (1)

b) Calculate the exact answer to 422 801, 15 ÷ 101 (2)

c) Work out the difference between your estimation and the calculated answer. (1)

6. a) Write down the 4th multiple of 18. (1)

b) Write down a multiple of 18 that is also a square number. (1)

7. Write the following ratios in their simplest form:

a) 150 minutes : $1\frac{1}{2}$ hours (1)

b) 150 cm : 1 060 mm (1)

8. Which ratio is larger, 18 : 5 or 72 : 25? (1)

9. A bell rings every 10 minutes and a buzzer sounds every 15 minutes. If the bell and buzzer both sound at 8 a.m., what is the next time that they will both go off at the same time? (2)

10. A coat costs R228 before VAT is added. What is the cost of the coat, including 14% VAT? Round off your answer to the nearest rand. (2)

11. In a recipe, 125 ml of milk is used for every $\frac{1}{2}$ ℓ of water. What is the ratio of milk to water? (2)

12. A survey found that 13 out of 108 men and 32 out of 233 women consult traditional healers for head-related ailments. Express these findings as a common fraction and as a decimal. Which group had the greater fraction? (3)

13. A car travelling at an average speed of 100 km/h covers a certain distance in 3 hours 20 minutes. At what constant speed must the car travel to cover the same distance in 2 hours and 40 minutes? (2)

14. R800 invested at r% per annum simple interest for a period of 3 years yields R168.

Use the formula $SI = \frac{Prt}{100}$ and calculate the value of r. (3)

15. Use the formula $A = P(1+\frac{r}{100})^n$ to calculate the compound interest at 7,5% per annum on a loan of R5 600 for 4 years.
(A = final amount) (2)

16. There are 9 single desks in a classroom and the rest of the desks are double desks. The classroom has seating for 33 learners. How many double desks are there in the room? (2)

17. If 6 gardeners can mow a lawn in 2 hours, how many gardeners can mow the same lawn in 3 hours? (2)

Total: 40 marks

Topic 2

Integers

Maths ideas

- Calculate with integers.
- Revise properties of integers.
- Calculate with squares, cubes, square roots and cube roots.

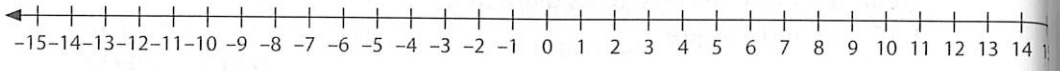
Key words

- **additive inverse** – the number you add to another number to get 0, for example –18 and 18 are additive inverses
- **positive integers** – whole numbers greater than 0
- **negative integers** – whole numbers less than 0

Calculations with integers

Every whole number has an opposite number called its **additive inverse**. A number and its additive inverse are the same distance away from zero. If you look at the number line below, you can see that every number has an opposite, except for zero.

Whole numbers greater than 0 are called **positive integers** and whole numbers less than 0 are called **negative integers**. The integer 0 is neutral. It is neither negative nor positive. We can call integers the set of whole numbers and their opposites. They can be illustrated on a number line with the negative integers to the left of zero and the positive integers to the right of zero.



Negative integers are always written with a negative sign. You read the negative sign as 'minus', for example you would read –14 as *minus* 14.

Positive integers are usually written without a positive sign. If a positive sign is written, you read the positive sign as 'plus', for example you would read +15 as *plus* 15.

Integers can be used in real-life situations, for example rising and falling temperatures, earning and spending money and stock market gains and losses.

Above sea level is the opposite of below sea level. Other opposites used in real-life problems are shown in the table.

positive	negative
increase	decrease
top	bottom
ascending	descending
forward	backward
right	left
smallest	largest

The mathematical symbol < means 'is less than' and > means 'is greater than'.

When you need to compare two integers or to arrange several integers in order, think of their relative positions on the number line.

The further to the *right* a number is on the number line, the *greater* its value.

The further to the *left* a number is on the number line, the *smaller* its value.

When we list numbers from smallest to largest we write the numbers in ascending order. When we write the largest number first and then the numbers from largest to smallest we write the numbers in descending order.

Example

1. Which number is larger, –120 or –102?
2. Arrange the following numbers in descending order:
   –126; 35; –215; 99; 0; –220

Answers

1. $-102 > -120$ ← –102 is further to the right on the number line
2. 99; 35; 0; –126; –215; –220 ← write the numbers from largest to smallest

EXERCISE 2.1

1. Which number is greater in these pairs?
   a) –716 405 or –393 910
   b) –11 050 or –22 110
2. Which number is smaller in these pairs?
   a) –78 319 or –79 361
   b) 0 or –1 010
3. Arrange these numbers in ascending order:
   a) 10 °C ; –9 °C; 8 °C; –12 °C; 0 °C
   b) R305; –R36; R192; –R70
4. Arrange these numbers in descending order:
   a) –17 °C ; –19 °C; 0 °C; –2 °C; 1 °C
   b) –R51 270; –R68 160; –R109 102; –R71 450
5. Calculate the value of the expression $3x^2 + 4x - 5$ when
   a) $x = -1$ b) $x = -2$ c) $x = 0$
6. The temperature in Moscow was 2 °C. Then it dropped to 8 °C below zero. By how many degrees did the temperature drop?
7. Which temperature is lower?
   a) –9 °C or –10 °C
   b) –25 °C or –52 °C
8. Mount Everest is 8 848 m above sea level and the Dead Sea is 400 m below sea level. What is the difference between the two elevations?
9. Mercury has a melting point of –39 °C and the freezing point of methanol is –98 °C. How much warmer is the melting point of mercury than the freezing point of methanol?
10. The Dow Jones average (a stock market share index) dropped from 12 837 to 12 503 in one week in June 2012.
    a) Calculate the drop in the share index.
    b) If the price continued to drop at the same rate, calculate the Dow Jones average after 4 more weeks.

Did you know?
15 135 120 can be divided exactly by any number from 1 to 16.

Challenge

Subtract the sum of the ninth to the sixteenth prime numbers from the sum of the first eight prime numbers.

Key words

- **like terms** – terms that are identical in respect of their variables
- **coefficient** – a number in front of a variable

Add and subtract terms with integer coefficients

You may only add or subtract **like terms** when simplifying an expression or solving an equation. Like terms have exactly the same variables and exponents. The number part of a term is called the **coefficient**, for example $-5x^2y$ and $2x^2y$ have integer coefficients –5 and 2 and the variable portion of each term is x^2y. They are like terms and may be added

$-5x^2y + 2x^2y = -3x^2y$. The coefficients of the variables are added. The variable part of each term remains unchanged.

$-4xy^2$ and $-4x^2y$ are not like terms as the variable part of the terms differ. These terms may not be added or subtracted.

The sum of two positive like terms is positive, for example
$+12xy + 34xy = +46xy$

When numbers are positive we do not have to write the '+' sign: $12xy + 34xy = 46xy$

The sum of two negative like terms is negative, for example
$(-22abc) + (-34abc) = -56abc$

We can also write this without the brackets as $-22abc - 34abc = -56abc$

To find the sum of two like terms with different signs, find the difference between the coefficients. The answer will have the sign of the coefficient with the greater numerical value (without the + or – signs), for example: $-48x^2 + 17x^2 = -31x^2$ but $48x^2 - 17x^2 = 31x^2$

EXERCISE 2.2

1. Calculate:

a) –73 – 110
b) –27 + 15
c) 49 – 62 – 12
d) –55 + 72 – 104
e) – 62 – 89 – 32
f) 43 – 12 – 105 + 70

2. Simplify the following:

a) $-15y + 6y$
b) $-109a - 21a$
c) $20ab - 31ab + 62ab$
d) $11a - 19b + 22a - 12b$
e) $-45x^2 + 37x + 14x^2 - 52x$
f) $-15a^2b - 19ab + 44a^2b - 61ab$

3. Write down the value of x that makes each of the following true:
   a) $-510 + 302 = x$
   b) $-150 + x = 0$
   c) $14 - 65 = -25 + x$
   d) $-86 - x = -110$
   e) $x - 2x = 35$
   f) $5x = -85$

4. Which of the following are like terms?
   a) $3bc$; $2ab$; $-12ac$; $8ab$
   b) x^2y; $-3xy$; $13x$; $12y$; $9xy$; $4xy^2$
   c) $11ab$; $10a^2b$; $-6ab^2$; $-a^2b$
   d) $2(3xy^2)$; $-4(5x^2y)$; $-6(4xy)$; $-x(xy)$
   e) $a(ab)$; $b(ab)$; $2a(ba)$; $-3b(2a)$
   f) $3x(2y^2)$; $2x(4y)$; $-5y(xy)$; $12y(3x^2)$

Multiply and divide terms with integer coefficients

In Grade 8 you learned that multiplying a positive integer with a negative integer results in a negative answer and that multiplying two negative integers results in a positive answer.

If you multiply two numbers with the ***same*** sign, the answer will be positive.
If you multiply two numbers with ***different*** signs, the answer will be negative.

Division is the inverse of multiplication so the same rules apply for dividing integers:

If you divide two numbers with the ***same*** sign, the answer will be positive.
If you divide two numbers with ***different*** signs, the answer will be negative.

Study the table of multiplication and division rules for integers:

$(+) \times (+) = +$	$(+) \div (+) = +$	multiplying or dividing two positive numbers gives a positive answer
$(+) \times (-) = -$	$(+) \div (-) = -$	multiplying or dividing a positive and a negative number gives a negative answer
$(-) \times (+) = -$	$(-) \div (+) = -$	multiplying or dividing a negative and a positive number gives a negative answer
$(-) \times (-) = +$	$(-) \div (-) = +$	multiplying or dividing two negative numbers gives a positive answer

Any terms may be multiplied or divided. Use the integer rules to multiply or divide the coefficients and use the exponent rules to multiply or divide the variables.

Example

Simplify:

a) $(-4x)(-3y)$

b) $(-24x^2y) \div (8xy^2)$

Answers

a) $(-4x)(-3y) = 12xy$ ← $(-4)(-3) = +12$ then multiply the variables

b) $(-24x^2y) \div (8xy^2)$

$= -\frac{3x}{y}$ ← $-24 \div 8 = -3$ and $\frac{x^2y}{xy^2} = \frac{x}{y}$

EXERCISE 2.3

1. Calculate the following:

a) -144×-20

b) $-125 \div -5$

c) -200×-30

d) $-190 \times -20 \div -100$

e) $-360 \div -60 \times 2$

f) $-96 \div -12$

2. Simplify the following:

a) $-15y \div 3y$

b) $-100a^2 \div 20a$

c) $2x^2 \times 3x \times -6x$

d) $-42xy^2 \div 7xy$

e) $-45x^2 \div 3x + 14x^2 \div 7x$

f) $35a^2b \times 2ab \div 7ab^2 - 10a^2$

3. Simplify:

a) $\frac{32abc}{64ac} \times \frac{8bc}{4b^2}$

b) $\frac{16x^2y}{2xy} \times \frac{24xy^2}{-12x}$

c) $\frac{12x}{y} \div \frac{6x}{y}$

d) $\frac{72mnp}{-9n} \div \frac{18mp}{36m^2}$

4. Find the values of the following:

a) $-5 \times 4 + 3 \times -6$

b) $12 \times -1 + 20 \times -2$

c) $16 \div -8 + 12 \div -3$

d) $-10 \times -7 - 22 \div -2$

e) $-2 \times 14 - 3 \times -5$

f) $84 \div -12 + 56 \div 8$

5. Simplify the following:

a) $-6a \times 4b + 2b \times -7a$

b) $11m \times -1n + 2m \times -6n$

c) $16y^2 \div -8y + 15y \div -5$

d) $-12q \times -6p - 18pq \div -3$

e) $-3ab \times 4bc - 2ac \times -6b^2$

f) $4mn \div -2m + 50n^2 \div 5n$

Mixed operations with integers

When you do a calculation that has mixed operations, the order of operations is sometimes important and can affect the answer. If you perform the calculations in the wrong order, you will get an incorrect answer.

When you add or multiply like terms the order of the numbers does not matter, for example: $-84x - 25x = -109x = -25x - 84x$ and $-12a \times -6a = 72a^2 = -6a \times -12a$

When numbers in brackets are multiplied by a number in front of the bracket, each number in the bracket is affected. This distributive property of numbers works for addition and subtraction, for example:
$-3(5a + 6a) = (-3 \times 5a) + (-3 \times 6a) = -15a + (-18a) = -33a$

If you simplified the brackets first, you would get the same answer:
$-3(5a + 6a) = -3(11a) = -3 \times 11a = -33a$

The correct order of operations is:

- First do calculations inside brackets, where possible.
- Next multiply and divide, working from left to right.
- Finally, add and subtract, working from left to right.
- You may only add and subtract constants or like terms.

EXERCISE 2.4

1. Multiply out the brackets and then simplify:

a) $-4(5a - 6b)$
b) $12(-10x + 5y)$
c) $-2(15m) + 3(-12m)$
d) $11(-7p) - 6(-8p)$
e) $2a(-3a + 12c)$
f) $-5b(6a - 5)$

2. Calculate:

a) $-18 + 6 \times (-3)$
b) $12 \div (-4) + 6$
c) $24 \times 3 - 20$
d) $100 \times (-2) + 50 \div (-5)$
e) $-10 \times 2 - 6 \div -3$
f) $-12 + 14 \div (-7) + 28 \div 4$

3. Simplify:

a) $12a(4a - 6b) + 9b(8a - 4)$
b) $(-2a \times 4b) - (-12a \times 6b)$
c) $-20(8x + 12y) - 8(12y - 10x)$
d) $-20m \times 8n + 20m \times (-12n)$
e) $2x(x - y) - 3y(y - x)$
f) $-3a(a - 2b) + 4b(2a - 1)$

4. Simplify:

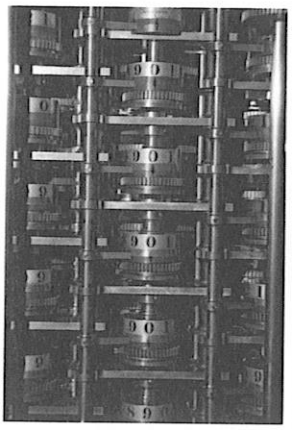
a) $y^2 - (-y)^2$
b) $2x(3x) - 3x(-2x)$
c) $-a(a + 1) - 2(3 - a) + a(a - 2)$
d) $-4m(2n^2) \div (-2n)^2 + 3m$
e) $3b(2c - 3d) - 6c(b + d) + 3d(b + c)$
f) $3xy(2 - x) - 2y(3x + 1) - y(-3x^2 - 2)$

Did you know?

The average surface temperature in Antarctica is –37 °C. Scientists predict that rising temperatures will cause ice shelves to break up and sea levels to rise.

Did you know?

One of the first mechanical calculating devices was devised by Charles Babbage who worked on it from 1821 to 1832. His "difference engine" was over 2 m tall and weighed about 15 tonnes. This was the first successful automatic calculator.



Squares, cubes, square roots and cube roots

Squares, cubes and powers

When any integer is squared, the answer is always positive, for example $(-12)^2 = (-12)(-12) = +144$ and $12^2 = 12 \times 12 = 144$

Note that two different numbers squared can give the same answer. This will be true for the square of any integer and the square of its additive inverse.

An integer raised to any even power will result in a positive answer:

$5^4 = 5 \times 5 \times 5 \times 5 = 625$

$(-5)^4 = (-5)(-5)(-5)(-5) = 625$

A power with an even exponent is always positive.

Note that $(-12)^2$ and -12^2 give different answers.

$(-12)^2 = 144$ as (-12) is squared $(-12)(-12) = 144$ but -12^2 is equal to $-(12)(12) = -144$.

In the case of -12^2, only the number 12 is squared. The minus sign is not inside a bracket, so is not affected by the power.

The minus sign is only squared when it is inside a bracket.

When any integer is cubed, the result has the same sign as the original number, for example $(-6)^3 = (-6)(-6)(-6) = -216$ but $6^3 = 6 \times 6 \times 6 = 216$

Cube numbers keep the sign of their cube root. The cube number -216 has the same sign as its cube root -6.

Any integer raised to an odd power will have an answer with the same sign as the original number.

A power with a negative base and an odd exponent is always negative.

For example: $(-3)^5 = (-3)(-3)(-3)(-3)(-3) = -243$

Example

Simplify:

a) $-8^2 + (-2)^6$ b) $\frac{(-9)^2}{-3^3}$

Answers

a) $-8^2 + (-2)^6$

$= -64 + 64$ ← use the rules to get the correct signs for each term

$= 0$

b) $\frac{(-9)^2}{-3^3}$

$= \frac{81}{-27}$ ← use the rules to get the correct signs for the numerator and denominator

$= -3$

EXERCISE 2.5

1. Calculate:

a) $(-2)^3 - 3^2$

b) $2^5 - (-2)^5$

c) $4^3 - (-3)^4$

d) $-1^3 - 2^3 - (-3)^3$

2. Simplify:

a) $\frac{(-4)^4}{2^6}$

b) $\frac{7^8}{-7^7}$

c) $\frac{(-5)^2}{-5^2}$

d) $\frac{-6^2}{(-3)^2(-2)^2}$

e) $\frac{(-2)^2(-5)^2}{10^2}$

f) $\frac{-3^3}{9} + \frac{8^2}{(-4)^2}$

3. Calculate:

a) $(-11)^2 + (-3)^3 - 9^2$

b) $3^3 - 2^3 - 5^2 + (-2)^2$

c) $-8^2 + (-2)^6 + (-6)^2$

d) $-1^7 \times (-2)^2 + 4^1$

e) $4^2 \div (-2)^4 + (-1)^{101}$

f) $(-12)^0 \times 6^2 - 4^3$

4. Calculate $2^{2014} - 2^{2013} \times 2$

Square roots and cube roots

In algebra, if you solve the equation $a^2 = 144$, there will be two correct answers for a.

For example $(-12)^2 = 144$ and $12^2 = 144$. The numbers are different but the result is the same for the square of both numbers.

The answer to the equation $a^2 = 144$ is written $a = \pm\sqrt{144}$ and the answers are 12 and −12. When there is no ± sign in front of a square root, there is only one answer and this is the positive square root.

When you solve the equation $\sqrt{144} = a$, there is only one answer: $a = 12$

$\sqrt{x}$ has only one positive answer for $x > 0$

There is no real solution to the square root of a negative integer. This is because there is no number that gives a positive answer when multiplied by itself.

Remember: **(+) × (+) = + and (−) × (−) = +**

You have already seen that any integer cubed results in an answer with the same sign as the base, for example $(-4)^3 = (-4)(-4)(-4) = -64$ but $4^3 = 4 \times 4 \times 4 = 64$

This means that $\sqrt[3]{-64} = -4$ and $\sqrt[3]{64} = 4$

A cube root with a negative base is always negative.

For example: $\sqrt[3]{-27} = \sqrt[3]{(-3)(-3)(-3)} = -3$

Example

Simplify:

a) $-\sqrt[3]{64} - \sqrt[3]{-27}$

b) $\dfrac{\sqrt[3]{-125}\sqrt{64}}{-\sqrt{100}\sqrt[3]{-64}}$

Answers

a) $-\sqrt[3]{64} - \sqrt[3]{-27}$

$= -4 - (-3)$ ← use the rules to get the correct signs for each term

$= -4 + 3$

$= -1$

b) $\dfrac{\sqrt[3]{-125}\sqrt{64}}{-\sqrt{100}\sqrt[3]{-64}}$

$= \dfrac{-5(8)}{-10(-4)}$ ← use the rules to get the correct signs for the numerator and denominator

$= \dfrac{-40}{40}$ ← simplify the numerator and denominator

$= -1$

EXERCISE 2.6

1. Simplify:

a) $\sqrt{81}$

b) $-\sqrt{121}$

c) $\sqrt[3]{-1}$

d) $\sqrt[3]{27}$

e) $\sqrt[3]{-64}$

f) $-\sqrt[3]{-125}$

2. Calculate:

a) $(-2)^3 + \sqrt[3]{-64}$

b) $\sqrt[3]{8} - (-1)^8$

c) $\sqrt{25} - (-5)^2 + \sqrt[3]{-27}$

d) $\sqrt[4]{16} + \sqrt[3]{-64} + \sqrt[5]{-32}$

3. True or false?

a) $\sqrt{25} - \sqrt{36} = \sqrt{-6}$

b) $\sqrt{4} \times \sqrt{9} = \sqrt{36}$

c) $\sqrt{-16} = -4$

d) $\sqrt[3]{-27} = -3$

4. Simplify:

a) $\sqrt{3}(\sqrt{3})^3$

b) $\sqrt{2}(\sqrt{2} - \sqrt{8})$

c) $\sqrt{49} - \sqrt[3]{-27} - \sqrt[3]{-64}$

d) $\sqrt{5}(\sqrt{5})^3 - \sqrt{3}(\sqrt{3})^5$

5. a) $\dfrac{\sqrt{81}\sqrt{64}}{\sqrt{16}\sqrt{9}}$

b) $\dfrac{\sqrt[3]{-64}\sqrt{81}}{\sqrt{100}\sqrt[3]{-27}}$

c) $\sqrt{49}\sqrt[3]{-27} \div (-3) + 23$

d) $2(\sqrt[3]{-64} + \sqrt{25})$

6. Simplify:

a) $\sqrt{144x^2y^2}$

b) $-\sqrt{\dfrac{x^4}{25}}$

c) $\sqrt[3]{-27y^3}$

d) $\sqrt{\dfrac{25x^2}{16y^2}}$

e) $\sqrt[3]{-\dfrac{64p^3}{8}}$

f) $-\sqrt{\dfrac{49}{m^2}} - \sqrt{\dfrac{81m}{m^3}}$

Challenge

Given the sum:
$(-2)^3 + (-3)^4 + (-4)^5 + (-5)^6$

Do you think the answer will be positive or negative?

Calculate the answer.

1. Write down the numbers that are integers from the following list of numbers:

$117{,}4\ ;\ -4\ 561\ 234\ ;\ \sqrt[3]{\frac{54}{2}}\ ;\ 67\ 598\ ;\ \frac{5}{8}\ ;\ 0{,}125\ ;\ -\frac{1}{2}\ ;\ \sqrt{36}$ (1)

2. Write the following numbers in ascending order:
$-9\ 942\ ;\ -16\ 432\ ;\ -33\ 915\ ;\ -20\ 020\ ;\ -8\ 978$ (1)

3. The temperature one night in Ulundi was measured at –1 °C. The temperature increased to 8 °C the next day. By how many degrees did the temperature rise? (1)

4. Which temperature is lower?
–3 °C or –4 °C (1)

5. Calculate the value of the expression $-x^2 + 3x - 2$ when
a) $x = -2$ b) $x = 0$ (2)

6. Calculate:
a) $-64 - 103 + 75$ b) $58 - (-17) + 12$ (2)

7. Simplify the following:
a) $16a^2 - 21b - 32a^2 + 19b$ b) $-5x^2 + 7x - 14x^2 - 32x$ (4)

8. Write down the value of x that makes each of the following true:
a) $-160 + x = 20$ b) $-2x = 48$ (2)

9. Calculate:
a) $-1\ 200 \times (-45)$ b) $-169 \div (-13)$
c) $32 - 14 \times (-3)$ d) $84 \times (-3) - 510 \div (-5)$ (4)

10. Simplify:
a) $(-15x)(-30y^2)$ (2)
b) $(-84x^2y) \div (12xy^2)$ (2)

11. Simplify:
a) $(-3)^3 - \sqrt[3]{-125}$ (2)
b) $\frac{-6a^2}{(3a)^2(-2a)^2}$ (2)
c) $-\sqrt{121a^2b^2}$ (2)
d) $\sqrt[3]{\frac{-27x^3}{64}}$ (2)

Total: 30 marks

Topic 3

Common fractions

Maths ideas

- Revise equivalent fractions and forms.
- Use all four operations with common fractions and mixed numbers.
- Use all four operations with numbers that involve the squares, cubes, square roots and cube roots of common fractions.
- Simplify algebraic fractions with common fractions as coefficients.
- Solve equations with common fractions as coefficients.
- Solve problems in context involving common fractions, mixed numbers and percentages.

Key words

- **common fraction** – a number written as $\frac{x}{y}$

Equivalent fractions and forms

A fraction is a portion of a whole or group that has been divided into equal parts. A **common fraction** is a number written as $\frac{x}{y}$. The number x is a whole number called the **numerator** and y is a non-zero whole number called the **denominator**.

We divide common fractions into two groups:

Proper fractions: The numerator of the fraction is less than the denominator, for example $\frac{6}{11}$.

Improper fractions: The numerator of the fraction is more than the denominator, for example $\frac{7}{3}$.

In some cases we write an improper fraction as a **mixed number**, for example we would write $\frac{8}{5}$ as $1\frac{3}{5}$. The mixed number has a whole number part and a fraction part.

Example

Convert:

a) $\frac{22}{7}$ into a mixed number

b) $5\frac{3}{4}$ into an improper fraction

Answer

a) $\frac{22}{7} = \frac{21}{7} + \frac{1}{7} = 3 + \frac{1}{7} = 3\frac{1}{7}$ ← we calculate $22 \div 7 = 3$ remainder 1. The remainder is the numerator of the fraction part of the mixed number.

b) $5\frac{3}{4} = 5 + \frac{3}{4} = \frac{5}{1} + \frac{3}{4} = \frac{20}{4} + \frac{3}{4} = \frac{23}{4}$ ← we can only add fractions if they have the same denominator.

The fractions $\frac{1}{8}$, $\frac{2}{16}$ and $\frac{125}{1\,000}$ are **equivalent fractions** because they have the same value. From previous grades we know that we can make an equivalent fraction by multiplying or dividing the denominator and the numerator by the same whole number, for example to add $\frac{1}{2}$ and $\frac{1}{6}$ we write $\frac{1}{2}$ in the its equivalent form $\frac{3}{6}$. We can now add $\frac{1}{2} + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$.

Always remember to write the fraction in its simplest form. To do this we need to find the highest common factor (HCF) between the numerator and the denominator. If you cannot remember how to do this refer back to Topic 1. If the HCF is not 1, the fraction is not in its simplest form. For example, $\frac{205}{15}$ is not in its simplest form. We simplify the fraction by dividing both the numerator and denominator by the HCF of 205 and 15, which is 5. Therefore:

$\frac{205}{15} = \frac{41}{3} = 41 \div 3 = 13$ remainder $2 = 13\frac{2}{3}$

It is sometimes necessary to convert common fractions into other forms.

Example

Write $\frac{4}{5}$ into its equivalent form:

a) as a decimal fraction b) as a percentage.

Answers

a) To write a common fraction as a decimal fraction, we first need to write the common fraction as an equivalent fraction with a power of 10 as denominator.

$\frac{4}{5} = \frac{8}{10} = 0{,}8$

b) To write a common fraction as a percentage, we need to change the fraction into its equivalent fraction with 100 as denominator.

$\frac{4}{5} = \frac{4 \times 20}{5 \times 20} = \frac{80}{100}$

$\frac{4}{5} = 80\%$

An easy way of finding the percentage form of a common fraction is to multiply the common fraction by 100.

EXERCISE 3.1

1. Write the following fractions and mixed numbers first as (i) decimal fractions and then as (ii) percentages:

   a) $\frac{3}{8}$ b) $\frac{64}{25}$ c) $1\frac{2}{5}$ d) $\frac{33}{50}$

2. Simplify the following common fractions and write as mixed numbers if an improper fraction:

   a) $-\frac{25}{15}$ b) $\frac{76}{1\,000}$

3. Write the common fractions in Question 2 as decimal numbers.

Challenge

Farmer Bongi counts his animals

A quarter of the animals on farmer Bongi's farm are pigs. $\frac{2}{7}$ of the animals are cows and one third are chickens. Determine which animal (pigs, cows or chickens) does farmer Bongi have the most of on his farm by using equivalent fractions.

Key words

- **numerator** – the whole number above the fraction line
- **denominator** – the whole number below the fraction line
- **proper fractions** – common fractions with the denominator bigger than the numerator
- **improper fractions** – common fractions with the denominator smaller than the numerator
- **mixed number** – a number consisting of a whole number and a fraction
- **equivalent fractions** – fractions with the same value

Key words

- **LCD** – lowest common denominator
- **reciprocal** – multiplicative inverse of a number, i.e. the number that you multiply a number by to get 1

Calculations with common fractions

Add and subtract common fractions

When adding and subtracting common fractions, we first need to make sure that the fractions have the same denominators. We first find the **lowest common denominator (LCD)** between all the denominators. Then we change all the fractions in the calculation to equivalent fractions with the LCD as denominator.

Example

Calculate:

$1\frac{3}{4}+\frac{1}{6}-\frac{1}{3}$

Answer

We need to first write all mixed numbers as improper fractions then find the LCD.

$\frac{7}{4}+\frac{1}{6}-\frac{1}{3}$

The LCD between 4, 6 and 3 is 12. We therefore have to change the three fractions into equivalent fractions with 12 as the denominator.

$\frac{21}{12}+\frac{2}{12}-\frac{4}{12}$

$=\frac{19}{12}$

$=1\frac{7}{12}$

Multiply and divide common fractions

When multiplying common fractions together, we simply multiply the denominators and numerators together. For example:

$\frac{4}{5}\times\frac{3}{8}=\frac{4\times3}{5\times8}=\frac{12}{40}=\frac{3}{10}$

When we multiply a common fraction by an integer we write the integer as a fraction with denominator of 1 and then multiply the common fractions together.

$5\times-\frac{3}{7}=\frac{5}{1}\times-\frac{3}{7}=-\frac{15}{7}=-2\frac{1}{7}$

When dividing a number by a common fraction, we multiply the number by the common fraction's **reciprocal**. For example:

$\frac{1}{4}\div\frac{3}{5}=\frac{1}{4}\times\frac{5}{3}=\frac{5}{12}$

When you divide, remember to invert and multiply.

Simplify algebraic expressions with common fractions as coefficients

When an algebraic expression contains common fractions as coefficients, we use the same calculation techniques as in the above sections to simplify the expressions if possible. There are some examples on page 31.

Example

Simplify:

a) $2ab + \frac{2}{3}cd - \frac{1}{2}cd + 1\frac{3}{4}ab$

b) $\frac{2}{5}x \times \frac{3}{4}x^2$

c) $1\frac{1}{4}a \div \frac{1}{2}$

d) $\sqrt{\frac{9}{16}y^4} + (\frac{1}{2}y)^2$

Answers

Always remember to change the mixed numbers into common fractions when doing your calculations.

Remember we can only add and subtract like terms.

a) $\frac{2}{1}ab + \frac{2}{3}cd - \frac{1}{2}cd + \frac{7}{4}ab$

$= \frac{8}{4}ab + \frac{4}{6}cd - \frac{3}{6}cd + \frac{7}{4}ab$

$= \frac{15}{4}ab + \frac{1}{6}cd$

b) We multiply the fractions together and then we multiply the variables.

$= \frac{2}{5} \times \frac{3}{4} \times x \times x^2$

$= \frac{6}{20}x^3$

$= \frac{3}{10}x^3$

c) $\frac{5}{4}a \div \frac{1}{2}$

$= \frac{5}{4}a \times \frac{2}{1}$

$= \frac{10}{4}a$

$= \frac{5}{2}a$

d) We calculate the square root of the first term and square the second term.

$= \frac{3}{4}y^2 + \frac{1}{4}y^2$

$= \frac{4}{4}y^2$

$= y^2$

EXERCISE 3.2

Check your answers using your calculator.

1. Calculate:

a) $\frac{1}{2} + 2\frac{3}{4} - \frac{3}{8}$

b) $\frac{9}{7} \times -\frac{3}{4} \div \frac{1}{3}$

c) $(\frac{2}{3})^3 - \frac{1}{2}$

d) $\sqrt[3]{\frac{27}{8}} \times \frac{5}{3}$

Did you know?

When we write $\frac{3}{4}x$ we are saying 3 divided by 4 multiplied by x. Because multiplication is commutative it is the same as saying 3 multiplied by x and then divided by 4. We can therefore write $\frac{3}{4}x$ as $\frac{3x}{4}$.

Thus: $\frac{3}{4}x = \frac{3x}{4}$

2. Simplify the following algebraic expressions:

a) $\frac{4}{5}x + \frac{1}{3}x - 1\frac{2}{3}x$

b) $\frac{9}{8}x \times 4$

c) $\frac{1}{4}xy \times \frac{8}{3}xy$

d) $\frac{5}{4}x \div \frac{4}{6}$

e) $(\frac{1}{2}x)^3 + \frac{3}{8}x^3$

f) $\sqrt{\frac{4}{9}x^2} + \sqrt[3]{27}x$

g) $1\frac{3}{5}xy \times \frac{2}{3}x - \frac{1}{4}x^2y$

h) $\frac{4}{3}x \times \frac{81}{16}x^3 \div \frac{3}{4}$

Solve equations

Algebraic equations often contain fractions as coefficients of terms. Consider the following examples before attempting the problems in the exercise.

Example

Solve for x:

a) $x + \frac{1}{2} = 2$

b) $\frac{5}{4}x = 1$

c) $2\frac{1}{7}x = \frac{1}{3}x + 38$

d) $2(x + \frac{3}{5}) = 4$

Answers

a) $x = 2 - \frac{1}{2}$

$x = \frac{4}{2} - \frac{1}{2}$

$\therefore x = \frac{3}{2}$

b) $x = 1 \div \frac{5}{4}$

$x = \frac{1}{1} \times \frac{4}{5}$ (multiplied by reciprocal)

$\therefore x = \frac{4}{5}$

c) Always change the mixed numbers into improper fractions.

$\frac{15}{7}x - \frac{1}{3}x = 38$

$\frac{45}{21}x - \frac{7}{21}x = 38$ (the LCD for 7 and 3 is 21)

$\frac{38}{21}x = 38$

$x = 38 \div \frac{38}{21}$

$x = \frac{38}{1} \times \frac{21}{38}$ (always write the integer as a fraction with denominator of 1)

$\therefore x = 21$

d) $2x + \frac{2}{1} \times \frac{3}{5} = 4$ (we remove the bracket and multiply the 2 with each term in the bracket)

$$2x + \frac{6}{5} = 4$$

$$2x = \frac{4}{1} - \frac{6}{5}$$

$$2x = \frac{20}{5} - \frac{6}{5} \text{ (the LCD of 5 and 1 is 5)}$$

$$2x = \frac{14}{5}$$

$$x = \frac{14}{5} \div \frac{2}{1} \text{ (remember to write the integer as a fraction with denominator of 1)}$$

$$x = \frac{14}{5} \times \frac{1}{2}$$

$$\therefore x = \frac{7}{5}$$

EXERCISE 3.3

Solve for :

1. $2x = \frac{1}{4}$
2. $-\frac{3}{4}x = 4$
3. $\frac{1}{3}x - 1 = 2$
4. $\frac{3}{5}x + 4 = 3$
5. $\frac{1}{4}x + \frac{5}{3} = 1$
6. $\frac{1}{7}x - \frac{1}{6}x = \frac{3}{4}$
7. $4(x + \frac{2}{3}) = -\frac{1}{2}$
8. $2(\frac{1}{2}x + 4) = \frac{1}{3}(\frac{3}{4}x + 1)$
9. $(\frac{2}{3})2x = \frac{1}{8}x + 2$
10. $\sqrt[3]{\frac{1}{8}x^3} = 4$

Remember to check your answers using your calculator. You can check your answer by substituting your answer back into the original equation to see whether it solves the equation.

Did you know?

Sometimes fractions are written with a diagonal fraction bar, ¼, ¾, ½. The diagonal fraction bar was introduced in the 1700s because the horizontal bar was difficult to use at a printing press. The earliest known usage of a diagonal fraction bar occurs in a hand-written document, where quantities of tea and coffee transactions are listed (¼ pound green tea). The earliest known printed instance of a diagonal fraction bar was in 1784.

Modified from:

http://www.scienceray.com/Mathematics/The-History-of-Fractions.271163

Solve problems

Often in real-life mathematical problems we deal with word problems involving fractions.

Example

Jack bought 10 oranges. On Monday he ate $\frac{2}{5}$ of them, on Tuesday $\frac{15}{50}$ and on Wednesday $\frac{6}{30}$.

a) What percentage of the oranges did he eat on Monday?
b) How many does he have left after Wednesday?

Answers

a) $\frac{2}{5} \times 100 = 40\%$

b) We need to first determine the fraction of the oranges he did not eat:

$= 1 - (\frac{2}{5} + \frac{15}{50} + \frac{6}{30})$ (Why do we subtract the sum of the fractions from 1?)

$= \frac{1}{1} - (\frac{60}{150} + \frac{45}{150} + \frac{30}{150})$

$= \frac{1}{1} - \frac{135}{150}$

$= \frac{10}{10} - \frac{9}{10}$

$= \frac{1}{10}$

He has $\frac{1}{10}$ of the 10 oranges left after Wednesday:

$= \frac{1}{10}$ of $10 = \frac{1}{10} \times \frac{10}{1} = 1$ orange

Challenge

A pharmacist mixes two chemicals to make cough medicine. The one chemical costs 19 cents per millilitre and the other costs 14 cents per millilitre. He mixes them in a ratio of 3 : 2 respectively. Find the selling price per millilitre if he makes a profit of 30%, in rand rounded to the nearest cent.

EXERCISE 3.4

1. The Smith family is going on a five day journey. On Tuesday they cover $\frac{7}{24}$ of the total distance, on Wednesday $\frac{11}{48}$, on Thursday $\frac{3}{16}$ and on Friday $\frac{1}{8}$.

a) What percentage of the Journey do they have to cover on Thursday?
b) What fraction of the total distance do they have to cover on Saturday?
c) The distance they will cover on the journey is 900 km. How many kilometres did they cover on Tuesday?

2. Lwandi got her Maths test back and was very happy about getting 85% for the test. To get full marks for the test she would need to get 60 marks. How many marks did she lose?

3. $\frac{4}{15}$ of a rugby team are injured and not able to play. Of those who are not injured, $\frac{1}{11}$ is sick and not able to play. What fraction of the players are able to play?

4. Sithe gave $\frac{8}{9}$ of his marbles to Tom and Jane. Of that, Jane got 0,25 of the marbles.

a) What fraction of the marbles did Sithe give to Jane?
b) What fraction of the marbles did Sithe give to Tom?

1. Convert the following common fractions and mixed numbers into (i) decimal fractions, (ii) percentages.

a) $\frac{5}{8}$ b) $\frac{17}{20}$ (2)

c) $3\frac{1}{5}$ d) $\frac{55}{1\,000}$ (2)

2. Calculate:

a) $\frac{8}{7} \times 4\frac{2}{3} + \frac{5}{6} \div \frac{6}{5}$ (2)

b) $(\frac{2}{3})^3 - \frac{1}{3} + \sqrt{\frac{4}{9}}$ (2)

c) $\sqrt[3]{2\frac{10}{27}} + \frac{5}{9}$ (2)

3. Simplify the following algebraic expressions:

a) $\frac{1}{7}x + \frac{2}{7}x - 1$ b) $\frac{4}{5}xy \div 3\frac{1}{5}$ (5)

c) $2\frac{1}{3}x \times (-\frac{7}{9}x) + \frac{1}{6}x^2$ d) $\sqrt{\frac{4}{25}x^2} \times (\frac{1}{2})^3$ (5)

4. Solve for x:

a) $\frac{1}{2}x - \frac{4}{5} = \frac{1}{3}$ (2)

b) $\frac{1}{2}(\frac{1}{3}x + \frac{1}{4}) = 2$ (3)

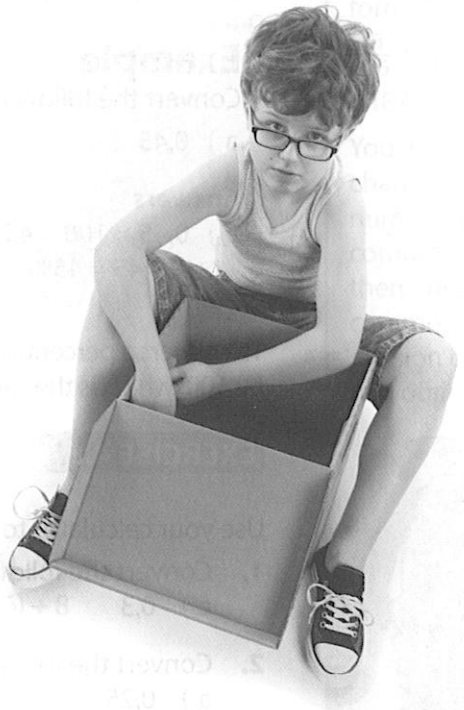
5. Convert the following percentages into common fractions:

a) 5% (1)

b) 125% (1)

c) 9,5% (1)

6. Josh and Jenny are in the same Mathematics class. Josh has completed $\frac{17}{24}$ of his homework and Jenny has completed $\frac{9}{13}$ of her homework. Who has completed more of the homework? (3)



7. $\frac{2}{3}$ of Ludwe's T-shirts are white. Of the white T-shirts $\frac{1}{6}$ have a pictures on them. What fraction of his white T-shirts have no picture on them? (3)

8. Jackson worked for 18 hours at the local shop. He worked $\frac{1}{3}$ of those hours packing the shelves, spending half of that time just opening packages. How many hours did he spend opening packages? (3)

9. The local soup kitchen has soup pots that hold 36 full cups of soup. The staff at the kitchen is instructed only to fill $\frac{3}{4}$ of each cup, so that people do not to spill their soup. How many $\frac{3}{4}$ full cups can they fill from one pot? (3)

Total: 40 marks