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C. DIRECT AND INDIRECT PROPORTION

DIRECT PROPORTION

Two quantities are **directly proportional** if as one quantity increases, the other increases. Or, as one quantity decreases, the other decreases.

If two quantities are **directly proportional** the ratio between the quantities remains constant.

Example 1: A school orders 52 calculators at a cost of R18 200.

- How much would 9 calculators cost?
- How many calculators can be purchased for R8 750?

a) Let the cost of 9 calculators be x .

Cost	Number of calculators
x	9
18 200	52

As the **ratio remains constant**, you can now set up an equation to solve for x :

$$\frac{x}{18\,200} = \frac{9}{52} \rightarrow \text{multiply both sides by } 18\,200$$

$$\begin{aligned}\therefore x &= \frac{9}{52} \times 18\,200 \\ &= \text{R}3150\end{aligned}$$



Always put whatever you are looking for (x) on top (as the numerator).

b) Let x be the number of calculators which can be purchased:

Number of calculators	Cost
x	8 750
52	18 200

As the **ratio remains constant**, you can now set up an equation to solve for x :

$$\frac{x}{52} = \frac{8\,750}{18\,200} \rightarrow \text{multiply both sides by } 52$$

$$\begin{aligned}\therefore x &= \frac{8\,750}{18\,200} \times 52 \\ &= 25 \text{ calculators}\end{aligned}$$

B. NEGATIVE EXPONENTS

Examine the Example below:

$$\frac{x}{x^2} = \frac{\cancel{x}}{\cancel{x} \times x} = \frac{1}{x}$$

It can be seen that $\frac{x}{x^2} = \frac{1}{x}$. However, from the laws of exponents we know that:

$$\frac{x}{x^2} = x^{1-2} = x^{-1}$$

From this we can conclude that $x^{-1} = \frac{1}{x^{+1}}$.

In other words, when a **base is raised to a negative exponent**, it means that **the base is on the wrong side** of the fraction line. This leads us into exponential laws 5 and 6 below.

LAW 5: A NUMERATOR RAISED TO A NEGATIVE EXPONENT

When a **base in the numerator is raised to a negative exponent**, it is equal to the **same base raised to a positive exponent** in the denominator.

$$a^{-x} = \frac{1}{a^{+x}}$$

LAW 6: A DENOMINATOR RAISED TO A NEGATIVE EXPONENT

When a **base in the denominator is raised to a negative exponent**, it is equal to the **same base raised to a positive exponent** in the numerator.

$$\frac{1}{a^{-x}} = a^{+x}$$

Example 1: Simplify the following, writing all bases with positive exponents:

- | | | | |
|-------------|-----------------------|-----------------------|----------------------------|
| a) x^{-3} | c) $5p^{-2}$ | e) p^7q^{-5} | g) $\frac{3}{2x^{-4}}$ |
| b) 2^{-4} | d) $\frac{x^{-5}}{8}$ | f) $\frac{1}{5^{-2}}$ | h) $\frac{y^{-3}}{x^{-4}}$ |

a) $x^{-3} \rightarrow$ move the base to the denominator
 $= \frac{1}{x^3}$

b) $2^{-4} \rightarrow$ move the base to the denominator
 $= \frac{1}{2^4}$
 $= \frac{1}{16}$

c) $5p^{-2} \rightarrow$ only the p is raised to a negative exponent
 $= \frac{5^1}{p^2} \rightarrow$ the 5 stays at the top as its exponent is positive

C. EXPONENTIAL EQUATIONS

An **exponential equation** is an equation where **the variable you are solving for is in the exponent**. When solving equations, it is important to note that if:

$$b^x = b^y \text{ then } x = y$$

This is known as **equating exponents**.

If the **bases are the same**, the **exponents are equal**.

The above rule is always true if the **base is not equal to 0 or 1**. 💡

STEPS TO SOLVING EXPONENTIAL EQUATIONS

- 1) **Make sure the exponent with the variable is alone on one side of the equation**
- 2) **Use exponent laws to get the same base on both sides of the equation**
- 3) **Equate the exponents on either side**

Example 1: Solve for x if $2^x = 2^8$.

$2^x = 2^8 \rightarrow$ as the bases are the same on both sides of the equation, the exponents are equal
 $\therefore x = 8$

Example 2: Solve for x if $3^x = 27$.

$3^x = 27 \rightarrow$ express 27 as power with base 3 to get the same bases on both sides
 $\therefore 3^x = 3^3 \rightarrow 27 = 3^3$
 $\therefore x = 3$

Example 3: Solve for x if $12^{x+5} = 144$.

$12^{x+5} = 144 \rightarrow$ express 144 as power with base 12 to get the same bases on both sides
 $\therefore 12^{x+5} = 12^2 \rightarrow 144 = 12^2$
 $\therefore x + 5 = 2 \rightarrow$ subtract 5 from both sides
 $\therefore x = 2 - 5$
 $\therefore x = -3$

Example 4: Solve for x if $5 \cdot 7^x = 245$.

$5 \cdot 7^x = 245 \rightarrow$ divide both sides by 5, the \cdot represents multiply
 $\therefore 7^x = 49 \rightarrow 245 \div 5 = 49$
 $\therefore 7^x = 7^2 \rightarrow 49 = 7^2$
 $\therefore x = 2$

Example 5: Solve for x if $5^x = \frac{1}{25}$.

$5^x = \frac{1}{25} \rightarrow$ express 25 as power with base 5 to get the same bases on both sides
 $\therefore 5^x = \frac{1}{5^2} \rightarrow$ bring the 5 to the top by making the exponent negative
 $\therefore 5^x = 5^{-2}$
 $\therefore x = -2$

Example 6: Solve for x if $31^x = 1$.

$31^x = 1 \rightarrow$ any number raised to the power of 0 equals 1
 $\therefore 31^x = 31^0 \rightarrow 31^0 = 1$
 $\therefore x = 0$

B. THE PRODUCT OF TWO BINOMIALS

When **two binomials** are multiplied by each other, **each term in first bracket has to be multiplied by each term in the second bracket**.

$$(a + b)(x + y) = ax + ay + bx + by$$

Example 1: Expand and simplify $(x + 2)(x + 3)$.

$$(x + 2)(x + 3)$$

$$= x^2 + 3x + 2x + 6 \rightarrow \text{add like terms together } (3x + 2x)$$

$$= x^2 + 5x + 6$$

Example 2: Expand and simplify $(x - 5)(x - 4)$.

$$(x - 5)(x - 4)$$

$$= x^2 - 4x - 5x + 20 \rightarrow \text{a '-' } \times \text{ '-' is a '+', add like terms together } (-4x - 5x)$$

$$= x^2 - 9x + 20$$

Example 3: Expand and simplify $(x - 7)(x + 2)$.

$$(x - 7)(x + 2)$$

$$= x^2 + 2x - 7x - 14 \rightarrow \text{a '-' } \times \text{ '+' is a '-', add like terms together } (+2x - 7x)$$

$$= x^2 - 5x - 14$$

Example 4: Expand and simplify $(3x - 5y)(2x + 2y)$.

$$(3x - 5y)(2x + 2y)$$

$$= 6x^2 + 6xy - 10xy - 10y^2 \rightarrow \text{a '-' } \times \text{ '+' is a '-', add like terms together } (+6xy - 10xy)$$

$$= 6x^2 - 4xy - 10y^2$$



Expanding the product of binomials is often taught as **FOIL**.
(**F**irst **O**uters **I**nners **L**ast)



Complete Exercise 2 on page 294.

SQUARE BINOMIALS

Square binomial are binomials in the form of: $(a+b)^2$ or $(a-b)^2$

Square binomials can be expanded as shown below:

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b) \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2\end{aligned}$$

$$\begin{aligned}(a-b)^2 &= (a-b)(a-b) \\ &= a^2 - ab - ab + b^2 \\ &= a^2 - 2ab + b^2\end{aligned}$$

It can be seen that a **general expression for squaring a binomial** is:

$$(\text{term 1} + \text{term 2})^2 = (\text{term 1})^2 + 2(\text{term 1} \times \text{term 2}) + (\text{term 2})^2$$

There always has to be a middle term when squaring a binomial. 💡

Example 1: Expand and simplify $(x-4)^2$.

$$(x-4)^2 = x^2 - 8x + 16$$

Example 2: Expand and simplify $(2x+3y)^2$.

$$(2x+3y)^2 = 4x^2 + 12xy + 9y^2$$

Example 3: Expand and simplify $(5x-3)^2$.

$$(5x-3)^2 = 25x^2 - 30x + 9$$

Example 4: Expand and simplify $(7x+1)^2$.

$$(7x+1)^2 = 49x^2 + 14x + 1$$

Example 5: Expand and simplify $(2a+11)^2$.

$$(2a+11)^2 = 4a^2 + 44a + 121$$



Complete Exercise 2 number 2 on page 294.


BINOMIALS WHICH DIFFER IN SIGN

Binomials which **differ in sign** are binomials in the form of: $(a - b)(a + b)$

In this case the two **brackets are exactly the same, except they differ in sign.**

The expansion of these type of binomials is shown below:

$$\begin{aligned}(a - b)(a + b) &= a^2 + ab - ab - b^2 \\ &= a^2 - b^2 \rightarrow \text{no middle term}\end{aligned}$$

In this case, the sum of the two middle terms is always equal to 0 and there is no middle term. 

Example 1: Expand and simplify $(x - 4)(x + 4)$.

$$\begin{aligned}(x - 4)(x + 4) \\ = x^2 - 16 \rightarrow (\text{term } 1)^2 - (\text{term } 2)^2\end{aligned}$$

Example 2: Expand and simplify $(2x - 3y)(2x + 3y)$.

$$\begin{aligned}(2x - 3y)(2x + 3y) \\ = 4x^2 - 9y^2 \rightarrow (\text{term } 1)^2 - (\text{term } 2)^2\end{aligned}$$

Example 3: Expand and simplify $(4x + 2)(4x - 2)$.

$$\begin{aligned}(4x + 2)(4x - 2) \\ = 16x^2 - 4 \rightarrow (\text{term } 1)^2 - (\text{term } 2)^2\end{aligned}$$

Example 4: Expand and simplify $(7x + 11)(7x - 11)$.

$$\begin{aligned}(7x + 11)(7x - 11) \\ = 49x^2 - 121 \rightarrow (\text{term } 1)^2 - (\text{term } 2)^2\end{aligned}$$

Example 5: Expand and simplify $(9x - 12y)(9x + 12y)$.

$$\begin{aligned}(9x - 12y)(9x + 12y) \\ = 81x^2 - 144y^2 \rightarrow (\text{term } 1)^2 - (\text{term } 2)^2\end{aligned}$$

Example 6: Expand and simplify $[(x - y) + 2][(x - y) - 2]$.

$$\begin{aligned}[(x - y) + 2][(x - y) - 2] \rightarrow (x - y) \text{ is term } 1 \\ = (x - y)^2 - 4 \rightarrow (\text{term } 1)^2 - (\text{term } 2)^2 \\ = x^2 - 2xy + y^2 - 4 \rightarrow \text{don't forget the middle term}\end{aligned}$$

Example 7: Expand and simplify $[(2a - b) + 3][(2a - b) - 3]$.

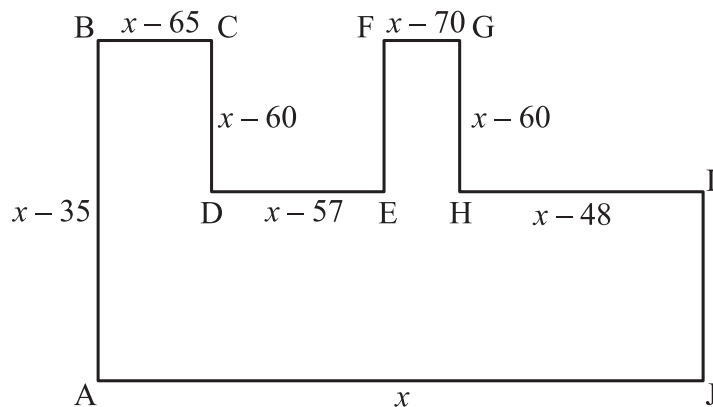
$$\begin{aligned}[(2a - b) + 3][(2a - b) - 3] \rightarrow (2a - b) \text{ is term } 1 \\ = (2a - b)^2 - 9 \rightarrow (\text{term } 1)^2 - (\text{term } 2)^2 \\ = 4a^2 - 4ab + b^2 - 9 \rightarrow \text{don't forget the middle term}\end{aligned}$$



Complete Exercise 2 number 3 on page 295.

Example 1: In the figure below all dimensions are in centimetres. Determine:

- The length of IJ.
- The value of x if the perimeter of the figure is 290cm.
- The area of the figure.



- $$IJ = AB - DC$$

$$= x - 35 - (x - 60) \rightarrow \text{don't forget to distribute the '-'}$$

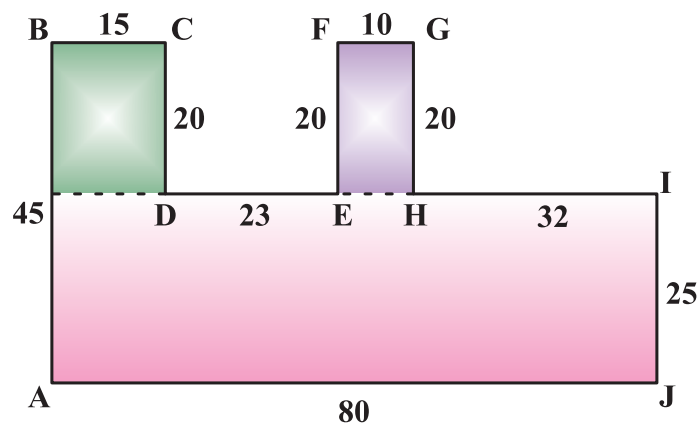
$$= x - 35 - x + 60$$

$$= 25 \text{ cm}$$
- $$P = x + x - 35 + x - 65 + x - 60 + x - 57 + x - 60 + x - 70 + x - 60 + x - 48 + 25$$

$$\therefore 290 = 9x - 430$$

$$\therefore -9x = -720 \rightarrow \text{divide both sides by } -9$$

$$\therefore x = 80 \text{ cm}$$
- To calculate the area of the shape you must break it up into 3 rectangles as shown in the diagram below:



$$A = (80 \times 25) + (15 \times 20) + (10 \times 20)$$

$$= 2000 + 300 + 200$$

$$= 2500 \text{ cm}^2$$

E. DETERMINING THE EQUATION OF A STRAIGHT LINE

There are cases in which you will be expected to determine the equation of a linear function:

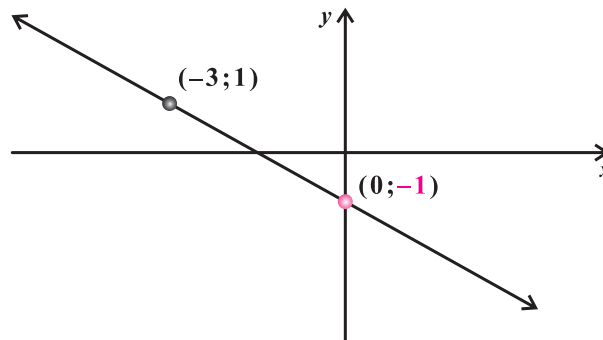
1. You will be given the **y-intercept** and one other point.
2. You will be given the coordinates of two points on the line.

CASE 1: GIVEN THE Y-INTERCEPT AND ONE OTHER POINT

In this case, substitute the **y-intercept** for **c** into the equation $y = mx + c$.

You then determine the value of **m** (the gradient) using the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Example 1: Determine the equation of the straight line shown below:



$$y = mx + c$$

$$\therefore y = mx - 1 \rightarrow -1 \text{ is the } y\text{-intercept}$$

You now have to calculate the gradient:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-1 - 1}{0 - (-3)} \\ &= -\frac{2}{3} \end{aligned}$$

$$\therefore y = -\frac{2}{3}x - 1$$

Example 2: Determine the equation of the straight line passing through the points **(0;5)** and **(-3;-1)**.

$$y = mx + c$$

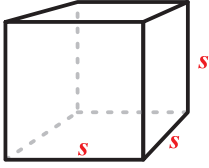
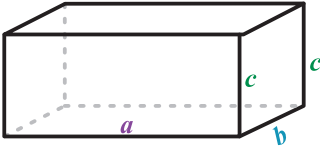
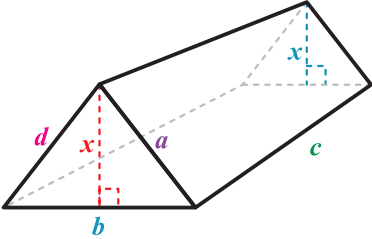
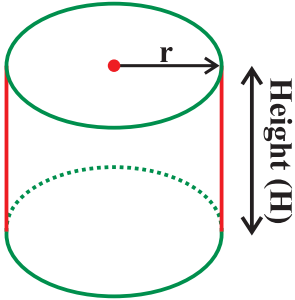
$$\therefore y = mx + 5 \rightarrow 5 \text{ is the } y\text{-intercept as the } x\text{-value at this point is } 0$$

You now have to calculate the gradient:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-1 - 5}{-3 - 0} \\ &= 2 \end{aligned}$$

$$\therefore y = 2x + 5$$

D. SUMMARY OF FORMULAS

Shape	Surface Area	Volume
<p style="text-align: center;">Cube</p>  <p style="text-align: center;">All sides are equal. This means that the area of all six faces are the same.</p>	$SA = 6(s)^2$	$V = (s)^3$
<p style="text-align: center;">Rectangular prism (cuboid)</p>  <p style="text-align: center;">There are a total of 6 faces. When calculating surface area add the area of each face together.</p>	$SA = 2(ab) + 2(ac) + 2(bc)$	$V = a \times b \times c$
<p style="text-align: center;">Triangular prism</p> 	$SA = ac + dc + bc + 2 \times \frac{1}{2}(b \times x)$ $= ac + dc + bc + bx$	$V = \frac{1}{2}(b \times x) \times c$
	<p>closed cylinder:</p> $SA = 2\pi rH + 2\pi r^2$ <p>cylinder open on one end:</p> $SA = 2\pi rH + \pi r^2$ <p>cylinder open on both ends :</p> $SA = 2\pi rH$	$V = \pi r^2 \times H$



Complete The Mixed Exercise on pages 363 and 364.

2. TWO-WAY TABLES

A two-way table is a way of representing every possible outcome of an experiment in table form.

Example 3: Two dice are rolled and the numbers on each face are written down.

- Draw a two-way table of all the possible outcomes.
- What is the probability of obtaining the same numbers on both dice?
- What is the probability of obtaining an even number on one dice and an odd number on the other?
- If the numbers on each dice are added together, what is the probability that the sum is greater than 12?

a)

		First Dice					
		1	2	3	4	5	6
Second Dice	1	1 ; 1	2 ; 1	3 ; 1	4 ; 1	5 ; 1	6 ; 1
	2	1 ; 2	2 ; 2	3 ; 2	4 ; 2	5 ; 2	6 ; 2
	3	1 ; 3	2 ; 3	3 ; 3	4 ; 3	5 ; 3	6 ; 3
	4	1 ; 4	2 ; 4	3 ; 4	4 ; 4	5 ; 4	6 ; 4
	5	1 ; 5	2 ; 5	3 ; 5	4 ; 5	5 ; 5	6 ; 5
	6	1 ; 6	2 ; 6	3 ; 6	4 ; 6	5 ; 6	6 ; 6

b) Probability = $\frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$

$$= \frac{6}{36} \rightarrow \text{there are 6 sets where the numbers are the same (orange in the table)}$$

$$= \frac{1}{6}$$

c) Probability = $\frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$

$$= \frac{18}{36} \rightarrow \text{there are 18 even and odd pairs (shown in green in the table)}$$

$$= \frac{1}{2}$$

d) Probability = $\frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$

$$= \frac{0}{36} \rightarrow \text{largest number that can be obtained is 12 when 6 and 6 are added}$$

$$= 0$$



Complete Exercise 2 on pages 395 and 396 and the Mixed Exercise on pages 396 and 397.

Binomials which Differ in Sign (from page 75)**3) Expand and simplify the following:**

a) $(x+3)(x-3)$

g) $(p-2)(p+2)$

m) $(2x-3y)(2x+3y)$

b) $(x-2)(x+2)$

h) $(a-4)(a+4)$

n) $(3ax-7by)(3ax+7by)$

c) $(x+7)(x-7)$

i) $(p-2q)(p+2q)$

o) $\left(\frac{2x}{3}-4y\right)\left(\frac{2x}{3}+4y\right)$

d) $(x-5)(x+5)$

j) $(r-3t)(r+3t)$

p) $\left(\frac{a}{3}-\frac{b}{2}\right)\left(\frac{a}{3}+\frac{b}{2}\right)$

e) $(x-11)(x+11)$

k) $(x-7y)(x+7y)$

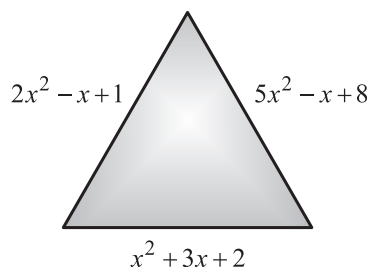
f) $(x+12)(x-12)$

l) $(w-9f)(w+9f)$

Solutions to Exercise 2 numbers 3a to n are on page 297 and numbers o and p are on page 298

Mixed Exercise:

- 1) The dimensions of a triangle are shown below. Determine the perimeter of the triangle. Your answer should be in terms of x .



Solutions to the Mixed Exercise number 1 are on page 298

2) Answer the following questions:

a) Arrange $2^3x^2 - 2^6x^4 - 5 + 2^9x$ in descending powers of x .

b) Subtract $4x^2 - 6x + 8$ from $-4x + 2$.

Solutions to the Mixed Exercise number 2 are on page 298

3) Expand and simplify the following:

a) $x(x-5)^2$

j) $(2b-4)^2 - (b+a)(b-a) - 3$

b) $(2a-b)(3a+5b)$

k) $\left(\frac{3}{7}x-4\right)\left(\frac{3}{7}x+4\right)$

c) $-5x(2x-1) - 2(x^2 - 3x + 5)$

l) $(x-3y)(x-7y) - (x-8y)^2$

d) $\left(\frac{3x^2}{5} - 7\right)\left(\frac{3x^2}{5} + 7\right)$

m) $x^2 - (x-4)(x+4)$

e) $3n - n(n-2) - 2(n-3) + n - 5n$

n) $4x - 5x\left(2x - \frac{1}{5}\right)$

f) $(2x-3)(3x+5)$

o) $(2a-5b)(3a-4b+6b)$

g) $3(3x-7)^2$

p) $p(2p+1)(3p-2) - 2p(p-7)^2$

h) $(4x-2)(2x-3)$

q) $(x^{2a} + 5)(x^{2a} - 5)$

i) $3\left(12x + \frac{2}{3}\right)^2$

Solutions to the Mixed Exercise number 3a to n are on page 298 and o to q are on page 299

4) Answer the questions below:

a) Use your calculator to determine the answer to $56874327^2 - (56874329)(56874325)$.

b) Expand and simplify $x^2 - (x+2)(x-2)$.

- c) The answer your calculator gives in 4a above is incorrect. Use your answer in 4b to prove that the calculator's answer is incorrect and determine the correct answer.

Solutions to the Mixed Exercise number 4 are on page 299

5) A swimming pool has a length of $x+12$ metres and breadth of $x-5$.

a) Determine an expression in terms of x for the area of the pool.

- b) A grade 9 learner suggests that x could be equal to 4 metres. Is this possible? Justify your answer.

Solutions to the Mixed Exercise number 5 are on page 299

Chapter 8 – Algebraic Expressions (Solutions)

Exercise 1 Solutions:

1)

a) 2

b) 2

c) 1

d) 2

e) 1

Exercise 1 number 1 is on page 294

2)

a) 3

b) -4

c) 12

d) 2

$$\begin{aligned} \text{e) } & 8(-1)^2 - 4(-1) + 12 \\ & = 8 + 4 + 12 \\ & = 24 \end{aligned}$$

Exercise 1 number 2 is on page 294

3)

$$\begin{aligned} \text{a) } & 8(x-3) - 3(2x-3y) \\ & = 8x - 24 - 6x + 9y \\ & = 2x + 9y - 24 \end{aligned}$$

$$\begin{aligned} \text{b) } & 2a(a-2b) - 3b(-a-5b) \\ & = 2a^2 - 4ab + 3ab + 15b^2 \\ & = 2a^2 - ab + 15b^2 \end{aligned}$$

$$\begin{aligned} \text{c) } & p(2-2q) - (p-5)(q) \\ & = 2p - 2pq - pq + 5q \\ & = 2p - 3pq + 5q \end{aligned}$$

$$\begin{aligned} \text{d) } & 2(3x^2 + 8x - 4) + x(x+3) \\ & = 6x^2 + 16x - 8 + x^2 + 3x \\ & = 7x^2 + 19x - 8 \end{aligned}$$

$$\begin{aligned} \text{e) } & (-2x)^2(x+3) - 5 + 3(2x+1) \\ & = 4x^2(x+3) - 5 + 6x + 3 \\ & = 4x^3 + 12x^2 + 6x - 2 \end{aligned}$$

Exercise 1 number 3 is on page 294

4)

$$\begin{aligned} \text{a) } & 2x^2 - 5x + 1 + x^2 - 5x + 1 \\ & = 3x^2 - 10x + 2 \end{aligned}$$

$$\begin{aligned} \text{b) } & -2x^2 + x + 1 - (3x^2 - 2x + 4) \\ & = -2x^2 + x + 1 - 3x^2 + 2x - 4 \\ & = -5x^2 + 3x - 3 \end{aligned}$$

$$\begin{aligned} \text{c) } & -5p^2 - 2q + 4 - (-2p^2 - q + 5) \\ & = -5p^2 - 2q + 4 + 2p^2 + q - 5 \\ & = -3p^2 - q - 1 \end{aligned}$$

Exercise 1 number 4 is on page 294

5)

$$\begin{aligned} \text{a) } & 8 - 5 \\ & = 3 \end{aligned}$$

$$\begin{aligned} \text{b) } & 5 - 3 \\ & = 2 \end{aligned}$$

$$\begin{aligned} \text{c) } & 3x^2 + 2x - 9 - (-5x^2 + 2x - 4) \\ & = 3x^2 + 2x - 9 + 5x^2 - 2x + 4 \\ & = 8x^2 + 0x - 5 \\ & = 8x^2 - 5 \end{aligned}$$

Exercise 1 number 5 is on page 294

Exercise 2 Solutions:

1)

$$\begin{aligned} \text{a) } & (x+3)(x+2) \\ & = x^2 + 2x + 3x + 6 \\ & = x^2 + 5x + 6 \end{aligned}$$

$$\begin{aligned} \text{c) } & (x-3)(x-2) \\ & = x^2 - 2x - 3x + 6 \\ & = x^2 - 5x + 6 \end{aligned}$$

$$\begin{aligned} \text{e) } & (2x+3)(x+1) \\ & = 2x^2 + 2x + 3x + 3 \\ & = 2x^2 + 5x + 3 \end{aligned}$$

$$\begin{aligned} \text{b) } & (x-3)(x+2) \\ & = x^2 + 2x - 3x - 6 \\ & = x^2 - x - 6 \end{aligned}$$

$$\begin{aligned} \text{d) } & (a+2b)(a-3) \\ & = a^2 - 3a + 2ab - 6b \end{aligned}$$

$$\begin{aligned} \text{f) } & (7x+1)(2x-3) \\ & = 14x^2 - 21x + 2x - 3 \\ & = 14x^2 - 19x - 3 \end{aligned}$$

Exercise 2 number 1 is on page 294 solutions are continued on the next page.